A Gain Factor Controlled SUMPLE Algorithm and System

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Abstract
In order to enhance signals with very low signal-to-noise ratio, a large array of sensors can be used to combine the received signals. SUMPLE, one of important signal combining approaches, uses less hardware and processing power while at the same time providing very low combining loss for weak signals. However, the combining loss of SUMPLE increases when a sensor in an array fails, and the weight amplitude of the sensor cannot drop to zero to automatically compensate for its failure. An enhanced SUMPLE algorithm is proposed in this paper. The gain factor of the explicit time delay and gain estimator is used to detect the failed sensor, and the corresponding weight amplitude is set to zero to minimize the combining loss. Mathematical model and performance analysis for a failed sensor in an array are presented. Simulation results indicate that the proposed algorithm performs better than the traditional state-of-the-art counterparts.

Index Terms
Signal combining, SUMPLE, gain factor, explicit time delay and gain estimator, weight amplitude.

1. Introduction

An array of sensors is an attractive technique for improving the reception of weak signals. One of the well-known methods is called signal combining, whose key step is to find a set of combining weight, so that the output signal-to-noise ratio (SNR) can be amplified by several folds. This technique is quickly used in different fields [1-14]. Until now, there have been two main approaches employed to obtain the combining weight. One of them, Eigen-based methods find the combining weight by estimating the signal and noise correlation matrices [1-4]. However, the Eigen-based methods endure the high computational cost especially for a large sensor array [4]. The other one, SUMPLE [5] obtains the combining weight by cross correlation of each sensor with the weighted sum of all the other sensors' output. SUMPLE reduces the amount of needed signal processing cost while at the same time providing comparable performance. Although many SUMPLE variants are proposed in [6-9], the combining performance of SUMPLE will decrease if a sensor in an array fails. To circumvent this drawback, a modified α coefficient SUMPLE (α-SUMPLE) is proposed in [10]. α coefficient can reduce the weight amplitude of the failed sensor in some degree, but incurs extra combining loss for an array without failed sensor.

As we all know, time delay synchronization between various sensors should be accomplished before signal combining. Adaptive time delay estimation (TDE) methods do not require a *priori* statistics about the source signals and their computational complexities are suitable for real time processing. The explicit time delay estimator (ETDE) [11] is an effective adaptive TDE algorithm and several ETDE variants are proposed in [12-14]. One of them, explicit time delay and gain estimator (ETDGE) [12] provides not only time delay but also gain factor, which can track the SNR of each sensor.

Therefore, a gain factor controlled SUMPLE (GFC-SUMPLE) algorithm and system are proposed in this paper. First, we introduce mathematical model and performance analysis for a failed sensor in an array. Then, we propose a unified system based on SUMPLE and ETDGE. In this system, the gain factor of ETDGE is used as the failure detection criterion. For this method, the weight amplitude of the failed sensor can be automatically set to zero, and no extra combining loss is incurred for an array without failed sensor.

The rest of this paper is organized as follow. In Section II, a problem formulation is introduced. The proposed algorithm is presented in Section III. Finally, simulation results and conclusions are included in Section IV and Section V, respectively.

2. Problem formulation

2.1 Basic definition

In the analysis and simulations presented in this study, it has been assumed that time delay synchronization between various sensor signals has been accomplished before signal combining [5]. The received signal from the ith sensor is represented by

$$S_{i,k} = s_{i,k} + n_{i,k}, \qquad (1)$$

where k is time index, $s_{i,k}$ is the source signal, and $n_{i,k}$ is the noise in this expression.

The combining weight is represented by

$$W_{i,K} = w_{i,K} + \eta_{i,K}, \tag{2}$$

where K is the time index in units of the correlation averaging interval, $ncor$. Moreover, $w_{i,k}$ is the signal weight, and $\eta_{i,K}$ is the noise weight.

Consequently, the combining output of the array is

$$C_k = \sum_{i=1}^{N} S_{i,k} W_{i,K}^*, \tag{3}$$

where N is the total number of sensors, and * represents complex conjugation.

The individual sensor signal SNR can be expressed as $\rho_s = |s_k|^2 / |n_k|^2$ and the weight SNR can be expressed as $\rho_w = |w_K|^2 / |\eta_K|^2$. The combining SNR is [5]

$$SNR = N\rho_s \left[\frac{\rho_w \rho + 1/N}{\rho_w + 1} \right], \tag{4}$$

where the ratio $\rho \approx 1$. From Eq.(4), it can be seen that the greater ρ_w, the better combining SNR. If ρ_w is far greater than 1, the combining SNR will be

$$SNR_{opt} = N\rho_s. \tag{5}$$

2.2 Related works

The weight for SUMPLE in the next time interval is [5]

$$W_{i,K+1} = \frac{R_{K+1}}{ncor} \sum_{k=K\cdot ncor}^{(K+1)ncor-1} \left[S_{i,k} \sum_{j=1, j\neq i}^{N} S_{j,k}^* W_{j,K} \right]. \tag{6}$$

The outer sum over $ncor$ corresponds to the correlation averaging interval, and the inner sum over all of the sensors except the ith sensor corresponds to the reference signal. The R_{K+1} is a normalization factor.

The weight for α-SUMPLE in the next time interval is [10]

$$W_{i,K+1} = \frac{R_{K+1} |W_{i,K}|^{\alpha}}{ncor} \sum_{k=K\cdot ncor}^{(K+1)ncor-1} \left[S_{i,k} \sum_{j=1, j\neq i}^{N} S_{j,k}^* W_{j,K} \right], \tag{7}$$

where $|W_{i,K}|^{\alpha}$ is a correcting coefficient which is used to reduce the weight amplitude of the failed sensor, and $\alpha \in [0,1]$.

3. GFC-SUMPLE algorithm

3.1 Failure analysis

Without loss of generality, we assume that $S_{1,K}$ only contains noise component. The combining output can be recast as

$$C_k = \sum_{i=2}^{N} s_{i,k} W_{i,K}^* + \sum_{i=1}^{N} n_{i,k} W_{i,K}^* = c_{k,N-1} + n_k^c, \tag{8}$$

where $c_{k,N-1}$ and n_k^c are the signal and noise components, respectively.

We can get ith sensor's signal weight $w_{i,K+1}$ and noise weight $\eta_{i,K+1}$ by substituting Eq.(8) into Eq.(6)

$$w_{i,K+1} = R_{K+1} s_{i,K} \left(c_{K,N-1}^* - s_{i,K}^* W_{i,K} \right), \tag{9}$$

$$\eta_{i,K+1} = \frac{R_{K+1}}{\sqrt{ncor}} \begin{bmatrix} \left(s_{i,K} + n_{i,K}\right)\left(n_K^c - n_{i,K}^* W_{i,K}\right) \\ + n_{i,K}\left(c_{K,N-1}^* - s_{i,K}^* W_{i,K}\right) \end{bmatrix}. \tag{10}$$

In Eq.(9) and Eq.(10), the averaged signal $s_{i,K}$ is equal to $s_{i,k}$ because $s_{i,k}$ is constant over time [5]. Due to $\rho_{wi} = |w_{i,K+1}|^2 / |\eta_{i,K+1}|^2$, we obtain [10]

$$\rho_{wi} = \frac{ncor \rho_s \rho_s - R_{K+1}^2 |s_0|^4 (1+\rho_s) \gamma_{wi}}{\rho_s + R_{K+1}^2 |s_0|^4 (1+\rho_s)\gamma_{wi}}, \tag{11}$$

where $|s_0|^2 = |s_{i,K}|^2$ and the power ratio γ_{wi} is

$$\gamma_{wi} = \frac{\left| \sum_{j=2, j\neq i}^{N} |W_{j,K+1}| \right|^2}{|W_{i,K+1}|^2} + \frac{|W_{1,K+1}|^2}{|W_{i,K+1}|^2}. \tag{12}$$

3.2 GFC-SUMPLE algorithm

From Eq.(12), we can see that the weight amplitude of the failed sensor has influence on the γ_{wi} of other sensors. Specifically, Eqs.(4), (11) and (12) demonstrate that if the weight amplitude of the failed sensor is not set to zero, the combining SNR of an array will decrease. Namely,

$$|W_{1,K+1}| \neq 0 \Rightarrow \gamma_{wi} \uparrow \Rightarrow \rho_{wi} \downarrow \Rightarrow SNR \downarrow. \tag{13}$$

Therefore, we consider using the gain factor of ETDGE to track the SNR of each sensor. If a gain factor is lower than a threshold, the corresponding weight amplitude will be set to zero. The system block diagram of ETDGE is shown in Figure 1.

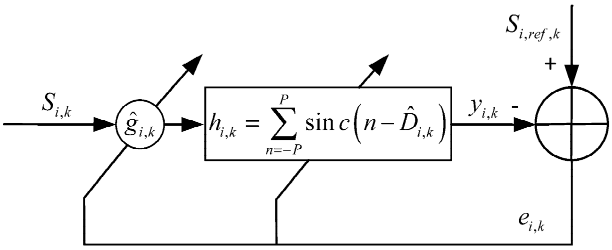

Figure 1. System block diagram of ETDGE

where $S_{i,ref,k}$ is a reference signal of $S_{i,k}$ and it can be selected from an array, the gain factor $\hat{g}_{i,k} = SNR/(1+SNR)$ [12], $\hat{D}_{i,k}$ is the time delay estimation and it can be used in the time delay synchronization, $h_{i,k}$ is the filter coefficients, and the filter length is 2P+1. The filter output signal is

$$y_{i,k} = \hat{g}_{i,k}\left({S_{i,k}}^T h_{i,k}\right), \tag{14}$$

where T represents transpose operator. The output error becomes

$$e_{i,k} = S_{i,ref,k} - y_{i,k}. \tag{15}$$

In this study, we mainly focus on the gain factor, which is updated according to [12]

$$\hat{g}_{i,k+1} = \hat{g}_{i,k} + \mu_g e_{i,k}\left({S_{i,k}}^T h_{i,k}\right), \tag{16}$$

where μ_g is the step-size of the gain factor.

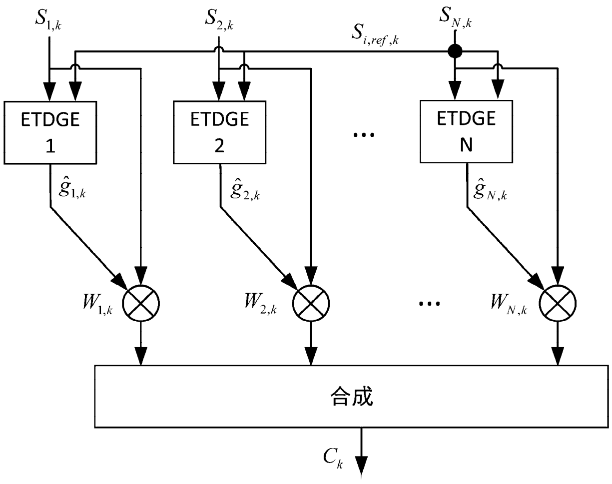

Figure 2. System block diagram of GFC-SUMPLE

Based on the above analysis, we propose a unified system based on SUMPLE and ETDGE. If the gain factor of ETDGE is lower than the threshold Δ, the weight amplitude of the corresponding sensor will be set to zero. Otherwise, the weight amplitude is unchanged as SUMPLE. The system block diagram of GFC-SUMPLE is shown in Figure 2.

4. Simulation results

To calculate the combining loss, the practical combining SNR is defined as

$$SNR_{prac} = \frac{\frac{1}{ncor}\sum_{k=K\cdot ncor}^{(K+1)ncor-1}\left(\sum_{i=1}^{N}\left|s_{i,k}W_{i,K}^{*}\right|^{2}\right)}{\frac{1}{ncor}\sum_{k=K\cdot ncor}^{(K+1)ncor-1}\left(\sum_{i=1}^{N}\left|n_{i,k}W_{i,K}^{*}\right|^{2}\right)}. \tag{17}$$

Then, the combining loss is

$$SNR_{loss} = SNR_{opt} - SNR_{prac}. \tag{18}$$

Simulations are conducted to compare the performance of SUMPLE, α-SUMPLE and GFC-SUMPLE for signal combining. The source is a uniform distribution random process, and $n_{i,k}$ is independent white Gaussian noises. α coefficient of α-SUMPLE is set to different values.

The aim of the test is to investigate the $\hat{g}_{i,k}$, the weight amplitude and the combining loss for an array when a sensor suddenly fails. To this end, we set $\mu_g = 0.0003$, 2P+1=21, $\Delta = 0.05$, N=10, $\rho_s = -10db$, K=40. The 1th sensor in the 10-sensor array drops out at K=20.

The simulation results are the statistics of 500 independent tests. The gain factor of 1th sensor is shown in Figure 3. We can find that the gain factor obviously decrease after K=20 and is lower than the threshold at K=25. It demonstrates that the gain factor can effective track the SNR of the sensor and find the failed sensor. The weight amplitude of 1th sensor is shown in Figure 4. We can see that the weight amplitude of the corresponding sensor is set to zero after K=25. The combining loss of 10-sensor array is shown in Figure 5. From Figure 5, as expected, the combining loss immediately increases to about 0.5dB when the sensor drops out. Although α-SUMPLE has better combining performance than SUMPLE after K=20, it also incurs more extra combining loss with greater α coefficient before K=20. What's more, GFC-SUMPLE can provide the better combining performance whether the sensor fails or not.

5. Summary

A gain factor controlled SUMPLE algorithm and system are proposed in this paper. We first introduced mathematical model and performance analysis for a failed sensor in an array. Then, we propose a unified system based on SUMPLE and ETDGE. In this system, the gain factor of ETDGE is used as the failure detection criterion. This algorithm can effectively find the failed sensor and the corresponding weight amplitude can be set to zero to minimize the combining loss. Moreover, GFC-SUMPLE, no extra combining loss is incurred for an array without failed sensor. Simulation results validate its effectiveness wrt the existing algorithms.

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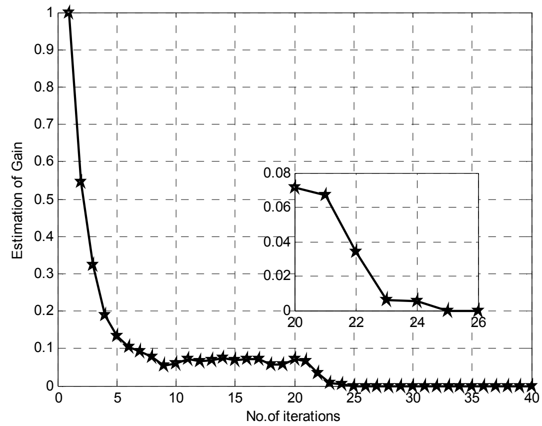

Figure 3. Gain factor for the failed sensor

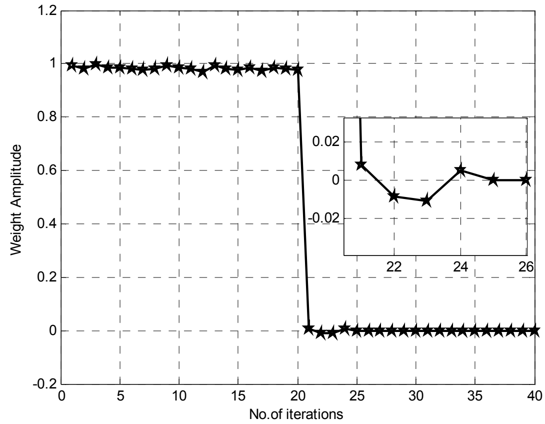

Figure 4. Weight amplitude for the failed sensor

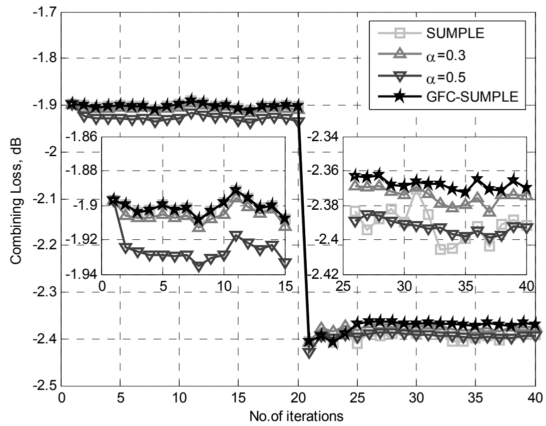

Figure 5. Combining loss for different algorithms as a single sensor drops out