

An Iteration-Based Variable Step-Size Algorithm for Joint Explicit Adaptation of Time Delay

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Abstract—By using an adaptive technique similar to the joint explicit time delay estimator, a novel variable step-size algorithm is derived. This algorithm employs the iteration time to control the step-size update and its respective performance analysis is also given. It is also proved that the proposed algorithm efficiently improves the convergence speed and the delay variance with only increasing a little computational cost compared to the conventional adaptive time delay estimation algorithm.

Index Terms—Signal combining, time delay estimation (TDE), variable step size (VSS).

I. INTRODUCTION

TIME delay estimation (TDE) between signals received at multiple spatially separated sensors has many important applications such as source location and direction finding [1]–[3]. The mathematical model is given by

$$x_i(k) = s(k + D_i) + n_i(k) \quad i = 1, 2, \dots, N \quad (1)$$

where $s(k)$ is the source signal, $n_i(k)$ are the uncorrelated white Gaussian noises with variance σ_n^2 , D_i is the time difference between the received signals, N is the number of received signals, and k is the sample index of signals.

Adaptive TDE methods do not require *a priori* statistics about the source signals and their computational complexities are suitable for real time processing. The explicit time delay estimator (ETDE) [4] is an attractive adaptive TDE algorithm. An improved version of ETDE has been proposed [5] and several ETDE variants have been compared [6]. Different to the above TDE algorithms, Luo and Yu [7] proposed a new joint explicit adaptation of time delay estimator (JETDE). This algorithm focuses on the joint adaptive time delay problem of multiple received signals and can effectively reduce the time delay variance, but its convergence speed is slow. All of these methods are fixed step-size (FSS) algorithms. But the FSS cannot simultaneously optimize the convergence speed and the delay variance. In other words, the FSS using larger step size

can achieve faster convergence speed but obtain worse delay variance, and vice versa.

Thus, a variable step-size (VSS) algorithm is expected to effectively balance between the convergence speed and the steady state estimation performance. Kwong and Johnston [8] used a power of instantaneous error to derive a VSS method. This method is quite sensitive to the noise disturbance. Aboulnasr and Mayyas [9] developed a method using an autocorrelation of errors to solve this problem. Huang *et al.* [10] derived an adaptive algorithm for blind single-input multiple-output systems. Luo *et al.* [11] presented a novel sigmoid function Luo variable step-size (LVSS) adaptive algorithm. References [12]–[14] introduced normalized least mean square (NLMS) and shrinkage NLMS methods. Mayyas and Momani [15] introduced a K. Mayyas variable step-size (KVSS) algorithm. Huang and Lee [16] also proposed a new variable step-size normalized (VSSN) algorithm that employs the mean square error (MSE) and the estimated system noise power to control the step-size update. These algorithms can alleviate the influence of noise disturbance in various degrees. However, for instance, $\mu(k+1) = \alpha\mu(k) + \gamma p^2(k)$ [9] [$\mu(k)$ is the step size and $p^2(k)$ is the error autocorrelation estimate, α and γ are parameters], although γ is normally chosen to be a very small value, the noise disturbance cannot be totally eliminated.

In this brief, an iteration-based VSS algorithm for JETDE (IVSS-JETDE) is proposed. This algorithm can provide fast convergence speed at early stages while ensure small delay variance at steady state. However, this algorithm is different from other VSS algorithms, since its step size is not controlled by the error signal but the iteration time. Thus, its performance is insensitive to the noise disturbance. It is proved that this algorithm can significantly improve the TDE performance.

The rest of this brief is organized as follows. In Section II, the related work of TDE algorithms is reviewed. The proposed algorithm is presented in Section III. Finally, simulation results and the conclusion is included in Sections IV and V, respectively.

II. RELATED WORK

A. ETDE

The ETDE employs the property that a delayed version of a band-limited signal can be expressed by convolving a sinc function with the signal itself. Therefore, this method and its variants are designed for narrow-band source signal. In fact, for most systems, the signals are band-limited because the

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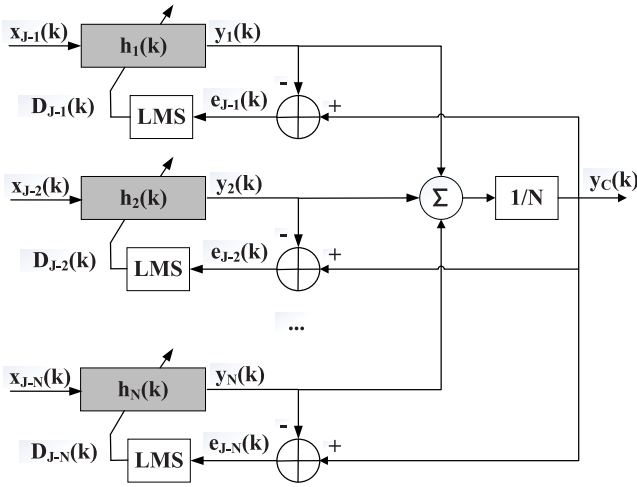


Fig. 1. System block diagram of the JETDE.

received signals are usually filtered before sampling, such as communication, radar, sonar, and so on. Basically, the ETDE has the same structure with the LMS TDE. However, the filter coefficients are replaced by $\text{sinc}(i - \hat{D}_e(k))$ with $\hat{D}_e(k)$ the delay estimate. The ETDE has a smaller delay variance than the LMS TDE especially for a short filter length, and the occurrence of false peak weights in the LMS TDE has a high probability at low signal-to-noise ratio (SNR) [6]. $\hat{D}_e(k)$ is updated iteratively according to [4]

$$\hat{D}_e(k+1) = \hat{D}_e(k) - 2\mu_e e(k) \sum_{i=-P}^P f(i - \hat{D}_e(k)) x(k-i) \quad (2)$$

$$f(v) = (\cos(\pi v) - \text{sinc}(v))/v \quad (3)$$

where $e(k)$ is the output error, μ_e is the step size of this algorithm, and $2P+1$ is the filter length. Taking the expected value of (2) yields

$$E\{\hat{D}_e(k)\} \approx D + (\hat{D}_e(0) - D) \left(1 - \frac{2}{3}\pi^2\sigma_s^2\mu_e\right)^k \quad (4)$$

where $\hat{D}_e(0)$ is the initial delay estimate, and σ_s^2 represents the signal power. If $0 < \mu_e < 3/(\pi^2\sigma_s^2)$, $E\{\hat{D}_e(k)\}$ will converge to delay value D . The delay variance, $\text{var}(\hat{D}_e)$, given by

$$\text{var}(\hat{D}_e) = \frac{6\mu_e\sigma_n^2(\sigma_s^2 + \sigma_n^2)}{\sigma_s^2[3 - \mu_e\pi^2(3\sigma_s^2 + \sigma_n^2)]}. \quad (5)$$

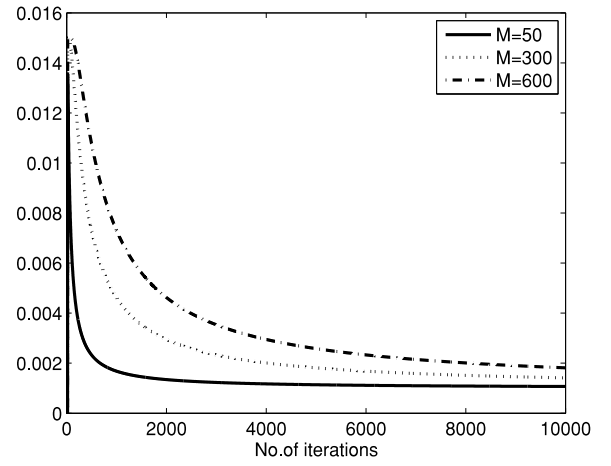
B. JETDE

The JETDE focuses on the joint adaptive time delay problem of N received signals and uses the combined signal as the reference signal. The system block diagram of the JETDE is shown in Fig. 1. The combined signal can be expressed as

$$y_c(k) = \frac{1}{N} \sum_{i=1}^N y_i(k). \quad (6)$$

The output error of the i th signal $e_{J-i}(k)$ thus becomes

$$e_{J-i}(k) = y_c(k) - y_i(k) = \frac{1}{N} \sum_{\substack{j=1 \\ j \neq i}}^N y_j(k) - \frac{N-1}{N} y_i(k). \quad (7)$$

Fig. 2. Learning characteristics of the $\mu_J(k)$.

The delay estimate $\hat{D}_{J-i}(k)$ is updated according to [7]

$$\begin{aligned} \hat{D}_{J-i}(k+1) = & \hat{D}_{J-i}(k) - 2\left(\frac{N-1}{N}\right)\mu_J e_{J-i}(k) \\ & \times \sum_{j=-P}^P f\left(j - \hat{D}_{J-i}(k)\right)x_{J-i}(k-j) \end{aligned} \quad (8)$$

where μ_J is the step size of this algorithm. The JETDE constrains the delay estimate of multiple received signals to align the average delay

$$\hat{D}_{J-i}(k+1) = \hat{D}_{J-i}(k+1) - \frac{1}{N} \sum_{j=1}^N \hat{D}_{J-j}(k+1). \quad (9)$$

As long as the condition $0 < \mu_J < 3N^2/(\pi^2\sigma_s^2(N-1)^2)$ is satisfied, the learning characteristics of the delay estimate can be expressed as

$$\begin{aligned} E\{\hat{D}_{J-i}(k)\} \approx & D_i + \{\hat{D}_{J-i}(0) - D_i\} \\ & \times \left(1 - \frac{2}{3}\pi^2\sigma_s^2\mu_J\left(\frac{N-1}{N}\right)^2\right)^k \end{aligned} \quad (10)$$

where D_i and $\hat{D}_{J-i}(0)$ are the same as D and $\hat{D}_e(0)$ in (4). Note that if μ_J is equal to μ_e , $(1 - 2/3\pi^2\sigma_s^2\mu_J((N-1)/N)^2) > (1 - 2/3\pi^2\sigma_s^2\mu_e)$. So the convergence speed of the JETDE is slower than that of the ETDE. The delay variance, $\text{var}(\hat{D}_J)$ can be expressed as

$$\text{var}(\hat{D}_J) = \frac{3\left(\frac{N-1}{N}\right)\mu_J\sigma_n^2(2\sigma_s^2 + \sigma_n^2)}{\sigma_s^2\left[3 - \mu_J\pi^2\left(\frac{N-1}{N}\right)^2(3\sigma_s^2 + \sigma_n^2)\right]} \quad (11)$$

since $\text{var}(\hat{D}_e)/\text{var}(\hat{D}_J) > 1$, the delay variance of the JETDE is smaller than that of the ETDE.

III. IVSS-JETDE

A. IVSS-JETDE

Similar to the JETDE, the IVSS-JETDE aims to solve the joint adaptive time delay problem. This section presents a

novel VSS algorithm providing fast convergence speed at early stages while ensuring small delay variance. This algorithm is different from other VSS LMS algorithms, since its step size is not controlled by the error signals but the iteration time. The relationship between the iteration time and the step size $\mu_I(k)$ can be expressed as

$$\mu_I(k) = \mu_{\min} + \mu \left[1 - e^{-\left(\frac{M}{k-m+1}\right)} \right] \quad (12)$$

where μ_{\min} is the minimum step size and its value is usually taken to be a very small value to ensure the minimum delay variance; μ is an initial step-size factor, and it is chosen to be a larger one in order to obtain a fast convergence speed; M represents the adjustment parameter that controls the changing speed of $\mu_I(k)$; and m is a start time and its initial value is 0. $\mu_I(0)$ and $\mu_I(\infty)$ can be written as

$$\mu_I(0) \approx \mu_{\min} + \mu = \mu_{\max} \quad (13)$$

$$\mu_I(\infty) \approx \mu_{\min} \quad (14)$$

where ∞ denotes infinite. Fig. 2 shows the learning characteristics of $\mu_I(k)$ with $\mu_{\min} = 0.001$ and $\mu = 0.007$ in (12) at different M value. It can be seen that the step size decreases with the increase of the iteration number. The IVSS-JETDE has a fast initial convergence speed due to its large step size. As the iteration number increases, the delay estimate will converge to its steady state and the step size will decrease gradually. The delay estimate $\hat{D}_{I-i}(k)$ is formulated as

$$\begin{aligned} \hat{D}_{I-i}(k+1) &= \hat{D}_{I-i}(k) - 2 \left(\frac{N-1}{N} \right) \mu_I(k) e_{I-i}(k) \\ &\quad \times \sum_{j=-P}^P f(j - \hat{D}_{I-i}(k)) x_{I-i}(k-j) \end{aligned} \quad (15)$$

where $e_{I-i}(k)$ is the same to $e_{J-i}(k)$ in (8). Comparing the IVSS-JETDE and the JETDE side-by-side, it can be seen that the updating equation of the delay estimate is the same for both methods, but μ_j is replaced by $\mu_I(k)$. Taking the expected value of (15) yields

$$\begin{aligned} E\{\hat{D}_{I-i}(k)\} &\approx D_i + \left\{ \hat{D}_{I-i}(0) - D_i \right\} \\ &\quad \times \left(1 - \frac{2}{3} \pi^2 \sigma_s^2 \mu_I(k) \left(\frac{N-1}{N} \right)^2 \right)^k \end{aligned} \quad (16)$$

where $\hat{D}_{I-i}(0)$ is the initial delay estimate. Provided that $0 < \mu_I(k) < 3N^2/(\pi^2 \sigma_s^2 (N-1)^2)$, $E\{\hat{D}_{I-i}(k)\}$ will converge to D_i . From (13) and (16), if $\mu_J < \mu_I(0)$, it can be seen that $(1 - 2/3\pi^2 \sigma_s^2 \mu_J((N-1)/N)^2) > (1 - 2/3\pi^2 \sigma_s^2 \mu_I(0)((N-1)/N)^2)$. This shows that the convergence speed of the IVSS-JETDE is faster than that of the JETDE. The delay variance, $\text{var}(\hat{D}_I)$, given by

$$\text{var}(\hat{D}_I) = \frac{3 \left(\frac{N-1}{N} \right) \mu_I(k) \sigma_n^2 (2\sigma_s^2 + \sigma_n^2)}{\sigma_s^2 \left[3 - \mu_I(k) \pi^2 \left(\frac{N-1}{N} \right)^2 (3\sigma_s^2 + \sigma_n^2) \right]}. \quad (17)$$

Similarly, if $\mu_J > \mu_I(\infty)$, $\text{var}(\hat{D}_J)/\text{var}(\hat{D}_I) > 1$, the delay variance of the IVSS-JETDE is smaller than that of the JETDE.

TABLE I
RELATIONSHIP OF THE STEADY STATE $P_{I-ie}(k)$
AND THE $p_{I-i}^2(k)$ AT DIFFERENT SNRS

SNR	0dB	2dB	4dB	6dB	8dB	10dB
$P_{I-ie}(k) <$	2.00	1.20	0.80	0.50	0.40	0.25
$p_{I-i}^2(k) <$	0.17	0.09	0.07	0.06	0.05	0.04

B. Tracking Ability

In order to make the IVSS-JETDE has the tracking ability, both the power of error signal $P_{I-ie}(k)$ and the error auto-correlation estimate $p_{I-i}^2(k)$ [9], [15] need to be estimated, namely

$$P_{I-ie}(k) = \frac{1}{2P+1} \sum_{j=k-(2P+1)+1}^k |e_{I-i}(j)|^2 \quad (18)$$

$$p_{I-i}^2(k) = (\beta P_{I-i}(k-1) + (1-\beta)e_{I-i}(k)e_{I-i}(k-1))^2 \quad (19)$$

where β is the same as those in [9] and [15]. The relationship of the steady state $P_{I-ie}(k)$ and the $p_{I-i}^2(k)$ at different SNRs are summarized in Table I.

It can be seen that based on the value of the steady state $P_{I-ie}(k)$, a hard threshold χ can be set. If $p_{I-i}^2(k) > \chi$, it indicates that the step size reaches to its μ_{\min} value much earlier or an abrupt change occurs in the unknown systems. Then, the step size of the IVSS-JETDE needs to immediately increase to μ_{\max} . Therefore, $m = k$ in (12) before use this updating equation. Note that $p_{I-i}^2(k)$ only be used to compare the threshold, and its step size is still controlled by the iteration time. Thus, this algorithm is different from the methods in [9] and [15].

To reduce computation, values of the exponential function in (12) are retrieved from pre-stored tables. For $P_{I-ie}(k)$, only needs to compute $|e_{I-i}(k)|^2$ at time k and other parts can use the results of the $P_{I-ie}(k-1)$. The computation requirements of the these adaptive delay estimations are summarized in Table II. The overall computational loads of the KVSS-JETDE and the VSSN-JETDE are much greater than the other methods. On the other hand, the IVSS-JETDE is a computationally efficient estimator, whose complexity is comparable to the ETDE and the JETDE.

IV. SIMULATION RESULTS

Simulations have been conducted to compare the performance of the ETDE, JETDE, LVSS-JETDE, KVSS-JETDE, VSSN-JETDE, and IVSS-JETDE for TDE. The sequences $s(k)$ and $n_i(k)$ are independent white Gaussian processes. The signal power is set to unity. And the IVSS-JETDE algorithm mainly focuses on the joint adaptive time delay problem of multiple received signals ($N > 2$).

To validate the effectiveness of the proposed method, the steady state MSE standard is adopted via $N_{\text{exp}} = 1000$ independent Monte Carlo runs. The MSE of delay estimation is defined as

$$\text{MSE}(k) = \frac{1}{N_{\text{exp}}} \sum_{j=1}^{N_{\text{exp}}} \left\| \hat{D}_j(k) - D \right\|_2^2. \quad (20)$$

TABLE II
COMPUTATIONAL COMPLEXITY COMPARISON OF THESE ADAPTIVE DELAY ESTIMATION METHODS

	Operation involved in algorithm			Additional requirements
	No. of additions	No. of multiplications	No. of divisions	
ETDE	6P+3	6P+5	0	one sinc and one cosine look-up operation
JETDE	6P+4+(2N-1)/N	6P+6	3/N	one sinc and one cosine look-up operation
LVSS-JETDE	6P+6+(2N-1)/N	6P+10	3/N+1	one sinc, one cosine and one exp look-up operation
KVSS-JETDE	8P+9+(2N-1)/N	8P+14	3/N+1	one sinc and one cosine look-up operation
VSSN-JETDE	8P+20+(2N-1)/N	8P+25	3/N+4	one sinc and one cosine look-up operation
IVSS-JETDE	6P+12+(2N-1)/N	6P+12	3/N+2	one sinc, one cosine and one exp look-up operation

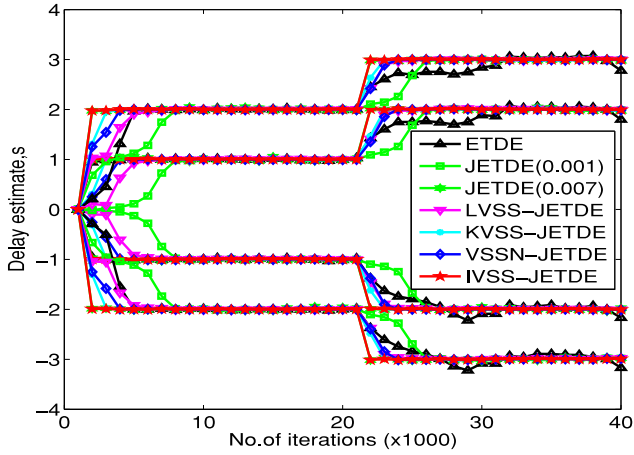


Fig. 3. Delay estimates at SNR = 10 dB with $N = 4$.

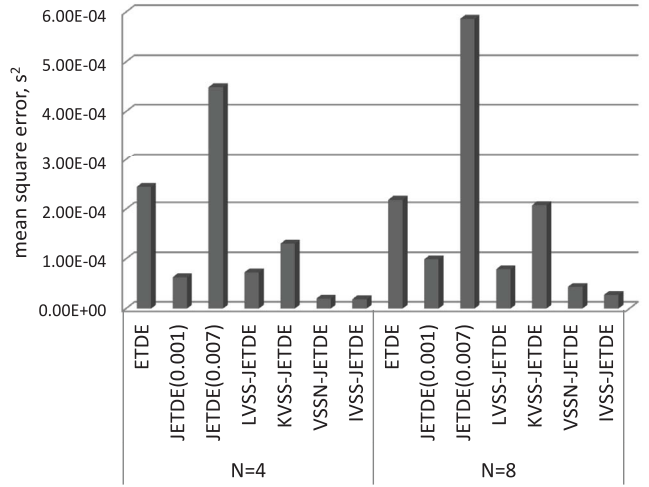


Fig. 5. Steady state MSE at SNR = 10 dB.

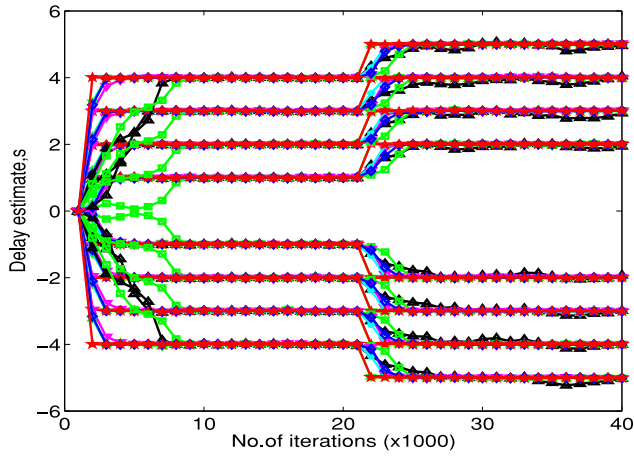


Fig. 4. Delay estimates at SNR = 10 dB with $N = 8$.

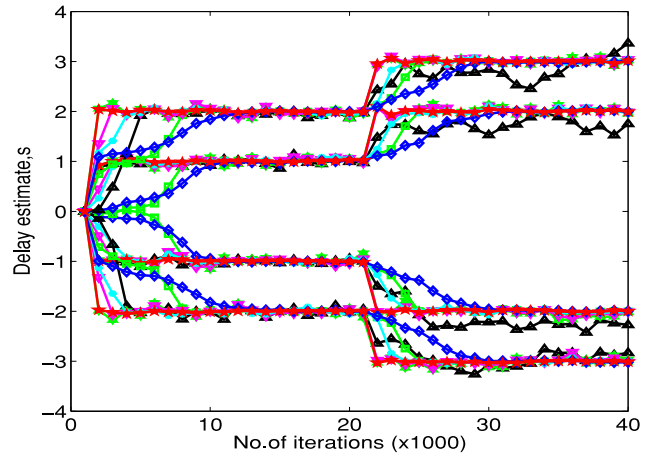


Fig. 6. Delay estimates at SNR = 0 dB with $N = 4$.

For $N = 4$, the delay values of these four received signals are 1, 2, -1, and -2 in the first 20000 iterations and $D = 2, 3, -2$, and -3 afterward. μ_e in (2) is selected as 0.001. μ_J in (8) is selected as 0.001 and 0.007. α and β of the LVSS-JETDE are chosen to be 1 and 0.01. α and δ of the KVSS-JETDE are chosen to be 0.98 and 0.025 [15]. α and β of the VSSN-JETDE are selected as 0.998 and 30 [16]. And the μ_{min} , μ and M of the IVSS-JETDE are chosen to be 0.000001, 0.007, and 200. The filter length $2P + 1$ is set to 21 and β in (19) is 0.98.

Fig. 3 shows the trajectories for the delay estimates of these algorithms at SNR = 10 dB. It can be seen that delay estimates

of these methods converge to the desired values at approximately the 2000th (IVSS-JETDE and JETDE($\mu_J = 0.007$)), 3000th (KVSS-JETDE), 4000th (VSSN-JETDE), 5000th (ETDE and LVSS-JETDE), and 7000th (JETDE($\mu_J = 0.001$)) iteration, respectively. At the same time, it can be observed that the IVSS-JETDE has the fastest tracking ability. In order to investigate the comparative performance for a different number of sensors, the previous test is repeated for $N = 8$ and the results are shown in Fig. 4. The delay values of these eight received signals are 1, 2, 3, 4, -1, -2, -3, and -4 in

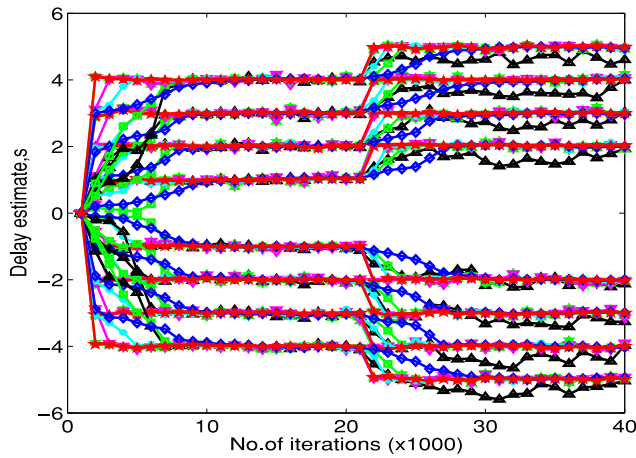


Fig. 7. Delay estimates at SNR = 0 dB with $N = 8$.

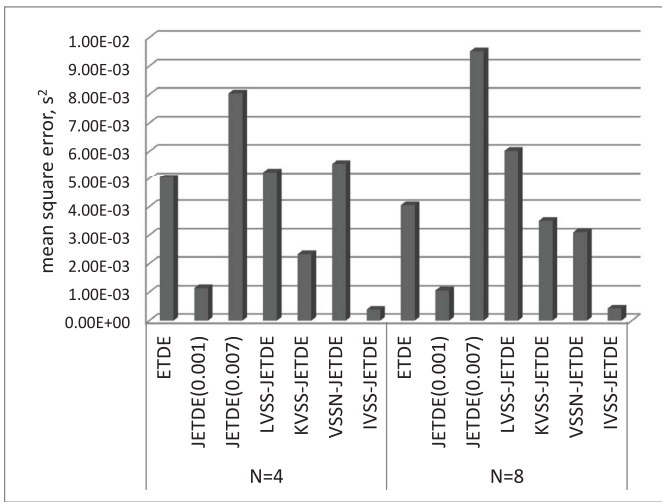


Fig. 8. Steady state MSE at SNR = 0 dB.

the first 20 000 iterations and $D = 2, 3, 4, 5, -2 - 3, -4$, and -5 afterward. Similarly, it can be seen that the IVSS-JETDE provides the fastest convergence speed and the tracking ability. The average steady state MSE of these methods at SNR = 10 dB are shown in Fig. 5. Although the JETDE($\mu_J = 0.007$) has almost identical convergence speed, it is the poorest delay estimator while the IVSS-JETDE provides the minimum delay variance. The MSE of the JETDE($\mu_J = 0.007$)($N = 4$), IVSS-JETDE($N = 4$), JETDE($\mu_J = 0.007$)($N = 8$), and IVSS-JETDE($N = 8$) are equal to 4.49×10^{-4} , 1.89×10^{-5} , 5.87×10^{-4} , and 2.76×10^{-5} , respectively.

The convergence characteristics of these algorithms for a step-changed delay at SNR = 0 dB for $N = 4$ and $N = 8$ are shown in Figs. 6 and 7, respectively, while their MSEs are plotted in Fig. 8. It can be seen that the delay estimates of the IVSS-JETDE converges to the desired values at approximately the 2000th.

From Fig. 8, it is observed that the IVSS-JETDE has the minimum delay variance 3.82×10^{-4} ($N = 4$) and 4.28×10^{-4} ($N = 8$). Again, the IVSS-JETDE still has the best convergence speed and delay variance. On the other hand, the convergence speed and delay variance of the other VSS algorithms obviously decrease. As a result, the IVSS-JETDE is also suitable to operate at a very low SNR environment.

V. CONCLUSION

In conclusion, this brief has derived the IVSS-JETDE algorithm whose step size is controlled by the iteration time. It is demonstrated that the IVSS-JETDE provides the better performance on the convergence speed and the delay variance at different SNRs with little computational cost.

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