Grid Optimization Based Methods for Estimating and Tracking Doubly Spread Underwater Acoustic Channels

Ye Qin
Key Laboratory of Information Technology for AUVs,
Institute of Acoustics, Chinese Academy of Sciences
University of Chinese Academy of Sciences
Beijing, China, 100190

Shefeng Yan, Zhuqing Yuan and Lijun Xu
Key Laboratory of Information Technology for AUVs,
Institute of Acoustics, Chinese Academy of Sciences
Beijing, China, 100190

***Abstract*—In this paper, the methods for estimating and tracking sparse doubly spread channels in single-carrier coherent communications are investigated. The sparse doubly spread channel is parameterized by a few paths with different delays, Doppler scales, and gains. Based on the model, a low-complexity channel estimation algorithm is proposed. The channel estimation is divided into two stages, the first for path delays, and the second for the corresponding residual Dopplers and gains. In either stage, parameters are estimated iteratively with the help of adaptive grid optimization, which can dramatically reduce computational complexity. We also propose a channel tracking algorithm, which takes advantage of the estimation result from the previous frame, to further reduce the complexity. Simulation results have demonstrated that the proposed method can achieve a comparable performance with much lower complexity compared to the existing two-stage approach with OMP.**

Keywords—sparse doubly spread channels; underwater acoustic channels; grid optimization; channel estimation; channel tracking; underwater acoustic communications

I. INTRODUCTION

To achieve high-speed and reliable communications in underwater mobile sensor networks of shallow water, the greatest challenge is undoubtly from the channel. The shallow water mobile acoustic channel is time-varying, and has significant multipath-Doppler doubly spread. All of these characteristics of the channel make it extremely difficult for the receiver to recover signals. Therefore, it is necessary to find an effective method to real timely estimate and track the channel state information (CSI), so that the receiver can compensate for the influence on the received signal caused by the poor channel conditions.

For estimating a shallow water mobile acoustic channel, the first problem to solve is to build a mathematical model to represent the channel. The simplest and most commonly used model to charaterize underwter acoustic (UWA) channel is the tapped-delay line model, but it is not appropriate for rapidly time-varying or doppler-spread channels. Related studies show that many shallow-water channels have sparse structures [1], the multipath arrivals of such channels can be resolved in delay. According to this, Li and Preisig [2] propose the delay-Doppler-spread function representation to characterize UWA channel. In this model, each channel path is assumned to have a constant Doppler shift, so such a representation can be regarded as a first-order approximation of the channel's rapid time variation. Under the representation, Li and Preisig [2] propose a delay-Doppler-spread function based sparse estimation approach, in which the dominant components on the delay-Doppler plane are identified via sparse estimation techniques. The greedy algorithms, including mathcing pursuit (MP) and othogonal matching pursuit (OMP), are used to search the parameters. It is the first time that compressed sensing algorithms are adopted in the estimation of UWA channels. In [3], a fast projected gradient method (FPGM) for estimating sparse doubly spread acoustic channels is proposed. An "l1-norm"constraint rather than the greedy seaching is adopted to estimate the sparse channel's delay-Doppler-spread function.

Although the delay-Doppler-spread function can characterize mobile UWA channel to some extent, it is not accurate enough, especially for the channel whose time variation is severe and complicated. Xu, Wang and Zhou [4] propose to use one polynomial to approximate the amplitude variation and another polynomial up to the first order to approximate the delay variation within a short period of time. Under such a channel parameterization, Qu and Nie [5] derive a discrete-time channel input–output relationship for single-carrier block transmissions, and propose a two-stage approach for the estimation of doubly spread acoustic channels. This approach can estimate the dynamic path parameters accurately and efficiently.

In this paper, we propose a novel method for estimating and tracking sparse doubly spread channels in single carrier underwater acoustic communications. With the help of the strategy of two-stage estimation [5] and the adaptive grid optimization technique [6], the new method can obtain an accurate estimate with quite low complexity. We also propose

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a channel tracking algorithm, which can further reduce the complexity by taking advantage of the previous estimate.

The rest of this paper proceeds as follows. The channel model and the corresponding representation of the discrete-time channel input–output relationship are given in Section II. Section III describes the proposed channel estimation and tracking method in detail. In section IV, simulation results are presented to verify the method's efficiency and performance. Finally, the paper is concluded in Section V.

II. SEYSTEM MODEL

Let $x(t)$ denote the transmitted signal, and $y(t)$ denotes the received signal. The channel input–output relationship is

$$\begin{aligned} y(t) &= x(t) * h(t,\tau) + v(t) \\ &= \int h(t,\tau)x(t-\tau)d\tau + v(t) \end{aligned} \tag{1}$$

where $v(t)$ is the ambient noise, $h(t,\tau)$ denotes the time-varying multipath channel impulse response which can be expressed as

$$h(t,\tau) = \sum_{p=1}^{N_{\mathrm{p}}(t)} A_p(t)\delta(\tau - \tau_p(t)) \tag{2}$$

where $A_p(t)$ and $\tau_p(t)$ are the amplitude and the delay of the p th path, respectively, and $N_{\mathrm{p}}(t)$ denotes the number of discrete paths that the channel contains. In a short period of time, $N_{\mathrm{p}}(t)$ can be assumed as a contant N_{p}.

Within one data frame's duration T_{f}, the time-varying amplitude of each path is approximated as a N_{a} th-order polynomial

$$A_p(t) \approx \sum_{n=0}^{N_{\mathrm{a}}} a_p^{(n)} t^n, \qquad t \in [0, T_{\mathrm{f}}]. \tag{3}$$

While the time-varying delay of each path is approximated as a first-order polynomial

$$\tau_p(t) \approx \tau_p - \alpha_p t, \qquad t \in [0, T_{\mathrm{f}}], \tag{4}$$

where τ_p is the initial delay, and α_p is termed as the Doppler scaling factor, which reflects the path's first-order dynamics, e.g., the platform is moving at a constant speed [5]. With such approximations, (2) can be rewritten as

$$h(t,\tau) \approx \sum_{p=1}^{N_{\mathrm{p}}} \sum_{n=0}^{N_{\mathrm{a}}} a_p^{(n)} t^n \delta(\tau - (\tau_p - \alpha_p t)), \tag{5}$$

and then the expression of the received signal in (1) can be derived

$$y(t) \approx \sum_{p=1}^{N_{\mathrm{p}}} \sum_{n=0}^{N_{\mathrm{a}}} a_p^{(n)} t^n x((1+\alpha_p)t - \tau_p) + v(t). \tag{6}$$

Applying the expression of the received signal (6), the discrete-time representation of the the channel imput-output relationship for single carrier coherent communication can be derived.

The transmitted coherent communication signal in passband is given by

$$x(t) = \mathrm{Re}\left\{ \sum_{k=1}^{K} s[k] w(t - (k - 1/2)T) e^{j2\pi f_c t} \right\}, \tag{7}$$

where $s[k]$ is the k th coherent communication symbol, T denotes the symbol duration, f_c is the carrier frequency and $w(t)$ is a rectangular window function

$$w(t) = \begin{cases} \frac{1}{T}, & -\frac{T}{2} < t \le \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}. \tag{8}$$

The receiver should first remove the dominant Doppler scale of the received signal $y(t)$ in passband by resampling with the estimated mean Doppler scaling factor $\hat{\alpha}$, leading to $r(t) = y(t/(1+\hat{\alpha}))$. Then the signal is shifted to baseband by removing the carrier, and filtering with a low pass filter. That is, $z(t) = \mathrm{LPF}\{r_{\mathrm{b}}(t) = r(t)e^{-j2\pi f_c t}\}$. Downsampling $z(t)$ at symbol rate, the mth sample (the sampling instant is set as $t = (m - 1/2)T$) can be expressed in detail as

$$\begin{aligned} z[m] = \sum_{k=1}^{K} s[k] \sum_{p=1}^{N_{\mathrm{p}}} & e^{j2\pi f_c(\beta_p(m-\frac{1}{2})T - \tau_p)} \times \\ & w\left((1+\beta_p)(m - \frac{1}{2})T - (k - \frac{1}{2})T - \tau_p\right) \\ & \cdot \sum_{n=0}^{N_{\mathrm{a}}} a_p^{(n)} \left(\frac{(m-\frac{1}{2})T}{1+\hat{\alpha}}\right)^n + v[m], \end{aligned} \tag{9}$$

where β_p is the residual Doppler factor of the pth path after resampling, which is defined as

$$\beta_p = \frac{\alpha_p - \hat{\alpha}}{1+\hat{\alpha}}. \tag{10}$$

Stack the transmitted symbols and received samples into vectors: $\mathbf{s} := [s[1], \cdots, s[K]]^T$ and $\mathbf{z} := [z[1], \cdots, z[M]]^T$. Then (9) can be written in a matrix-vector formula for simplicity

$$\mathbf{z} = \mathbf{H}\mathbf{s} + \mathbf{v}, \tag{11}$$

where $\mathbf{v}$ is the vector of ambient noise, and

$$\mathbf{H} = \sum_{p=1}^{N_{\mathrm{p}}} \sum_{n=0}^{N_{\mathrm{a}}} a_p^{(n)} \mathbf{\Phi}_p^{(n)} \mathbf{W}_p \tag{12}$$

is the channel matrix, in which $\mathbf{\Phi}_p^{(n)}$is an $M \times M$ diagonal matrix with the element

$$\left[\mathbf{\Phi}_p^{(n)}\right]_{m,m} = e^{j2\pi f_c(\beta_p(m-\frac{1}{2})T-\tau_p)}\left((m-\frac{1}{2})T\right)^n, \quad (13)$$

and $\mathbf{W}_p^{(n)}$ is an $M \times K$ matrix with the element

$$[\mathbf{W}_p]_{m,k} = w\left([(1+\beta_p)(m-\frac{1}{2})-(k-\frac{1}{2})]T-\tau_p\right). \quad (14)$$

III. Doubly Spread Channel Estimation Methods

The discrete-time representation of the the channel imput-output relationship for single-carrier coherent communications in time-varying doubly spread channels is given in section II (see (9), (11) and (12)). With the model, the channel estimation becomes the problem to estimate the parameter triplets $\left\{\{a_p^{(n)}\}_{n=0}^{N_\mathrm{a}}, \tau_p, \beta_p\right\}_{p=1}^{N_\mathrm{pa}}$ via the received signals in baseband and the known training sequence, so that we can use these parameters to reconstruct a channel matrix $\mathbf{H}$ to recover the unknown transmitted symbols. Accordingly, we establish the measurement equation for channel estimation

$$\mathbf{z}_\mathrm{t} = \mathbf{H}\mathbf{s}_\mathrm{t} + \mathbf{v}_\mathrm{t} = \sum_{p=1}^{N_\mathrm{p}}\sum_{n=0}^{N_\mathrm{a}} a_p^{(n)}\mathbf{\Phi}_p^{(n)}\mathbf{W}_p\mathbf{s}_\mathrm{t} + \mathbf{v}_\mathrm{t}, \quad (15)$$

where $\mathbf{v}_\mathrm{t}$ denotes the vector of measurement noise, $\mathbf{s}_\mathrm{t}$ and $\mathbf{z}_\mathrm{t}$ denote the vector of training symbols and the corresponding observation vector , respectively. For sparse channels, the number of paths N_p is quite small compared to the dimension of $\mathbf{H}$, so it is appropriate to use compressed sensing algorithms to solve the measurement equation with limited number of measurements. The most direct way to solve the problem is to construct a dictionary containing all possible combinations of (τ_p, β_p), and find the ones that match the observation vector best. However, this method is too difficult for pratical use, because the amount of possible combinations is usually very large, leading to the extremely high column dimension of the dictionary. In that case, we have to use a long observation vector to solve the measurement equation, making the computational complexity too high. So it's necessary to find a way to lower the dimension of the dictionary for estimation.

A. Two-stage Strategy

The first strategy we adopt to lower the dimension of the dictionary for estimation is to divide the estimation into two stages, searching for τ_p and β_p in each stage, respectively and sequentially, rather than searching the two-dimensional parameter pair (τ_p, β_p) in one stage. This strategy, termed as two-stage approach [5], turns the estimation from a two-dimensional searching problem into a one-dimensional searching problem. The brief description of the strategy is as follows.

For a frame of symbols, one training sequence is inserted at the beginning of the frame (in front of the data sequence), and another at the back of the frame (behind the data sequence). The channel estimation of the frame is devided into two stages.

For the first stage, within the duration of the preceding training sequence, m is so small that β_p has little influence on the phase shift of the sample, so we can assume that $\beta_p = 0$, and try to find the right initial delays on the given grid

$$\mathcal{T} = \left\{0, \frac{\tau_\mathrm{max}}{\lambda}, \frac{2\tau_\mathrm{max}}{\lambda}, ..., \tau_\mathrm{max}\right\}$$

with the resolution $\tau_\mathrm{max}/\lambda$, where λ is an integer, defined as the resolution factor of the desighed delay grid. Then the estimation problem can be fomulated as

$$\begin{aligned}\mathbf{z}_\mathrm{f} &= \sum_{\tau\in\mathcal{T}} a_\tau\mathbf{\Phi}_\tau^{(0)}\mathbf{W}_\tau\mathbf{s}_\mathrm{f} + \mathbf{v}_\mathrm{f} \\ &= \mathbf{D}_\mathcal{T}\mathbf{a}_\mathrm{f} + \mathbf{v}_\mathrm{f},\end{aligned} \quad (16)$$

where $\mathbf{s}_\mathrm{f}$ denotes the front training sequence, and

$$\mathbf{D}_\mathcal{T} = \left[\mathbf{\Phi}_{\tau_1}^{(0)}\mathbf{W}_{\tau_1}\mathbf{s}_\mathrm{f}, ..., \mathbf{\Phi}_{\tau_{N_\mathcal{T}}}^{(0)}\mathbf{W}_{\tau_{N_\mathcal{T}}}\mathbf{s}_\mathrm{f}\right]$$

is the dictionary whose columns correspond to the initial delays on $\mathcal{T}$,. and $\mathbf{a}_\mathrm{f} = [a_{\tau_1}, ..., a_{\tau_{N_\mathcal{T}}}]^T$ denotes the zeroth-order polynomial coefficients of the corresponding path gains. Searching over $\mathcal{T}$ with OMP, we can obtain the estimated initial delays $\hat{\tau}_1, ..., \hat{\tau}_{N_\mathrm{p}}$.

For the second stage, within the duration of the back training sequence, construct the residual Doppler factor grid

$$\mathcal{B} = \left\{-\beta_\mathrm{max}, -\beta_\mathrm{max} + \frac{\beta_\mathrm{max}}{\xi}, -\beta_\mathrm{max} + \frac{2\beta_\mathrm{max}}{\xi}, ..., \beta_\mathrm{max}\right\}$$

with the resolution β_max/ξ based on the estimated initial delay grid $\mathcal{T}' := \{\hat{\tau}_1, ..., \hat{\tau}_{N_\mathrm{p}}\}$, and ξ is an integer, defined as the resolution factor of the given Doppler grid. Assuming that one platform is moving at a constant speed for simplicity, we just need to take the zeroth's path gains into account, and then the fomulation of the estimation problem can be expressed as

$$\begin{aligned}\mathbf{z}_\mathrm{b} &= \sum_{\tau\in\mathcal{T}',\beta\in\mathcal{B}} a_{\tau,\beta}\mathbf{\Phi}_{\tau,\beta}^{(0)}\mathbf{W}_{\tau,\beta}\mathbf{s}_\mathrm{b} + \mathbf{v}_\mathrm{b} \\ &= \mathbf{D}_{\mathcal{T}',\mathcal{B}}\mathbf{a} + \mathbf{v}_\mathrm{b}\end{aligned} \quad (17)$$

where $\mathbf{s}_\mathrm{b}$ denotes the back training sequence, and

$$\begin{aligned}\mathbf{D}_{\mathcal{T}',\mathcal{B}} &= \left[\mathbf{\Phi}_{\tau_1,\beta_1}^{(0)}\mathbf{W}_{\tau_1,\beta_1}\mathbf{s}_\mathrm{b}, ..., \mathbf{\Phi}_{\tau_{N_\mathrm{p}},\beta_{N_\mathcal{B}}}^{(0)}\mathbf{W}_{\tau_{N_\mathrm{p}},\beta_{N_\mathcal{B}}}\mathbf{s}_\mathrm{b}\right], \\ \mathbf{a} &= \left[a_{\tau_1,\beta_1}, \ ... \ , a_{\tau_{N_\mathrm{p}},\beta_{N_\mathcal{B}}}\right]^T.\end{aligned}$$

Searching over $\mathcal{B}$ with OMP, we can obtain the estimate of the residual Doppler factors $\hat{\beta}_1, ..., \hat{\beta}_{N_\mathrm{p}}$ and the estimate of polynomial coefficients of the corresponding path gains $\hat{a}_1, ..., \hat{a}_{N_\mathrm{p}}$.

With the two-stage strategy, the column dimension of the dictionary is reduced by a large margin, leading to a significant reduction in the observation length. Besides, the strategy can avoid the interference between the parameters, resulting in more accurate estimate [5]. So we can obtain a better performance with much lower complexity compared to the method using OMP in only one stage.

B. Grid Optimization based Channel Estimation

In either stage of the aforementioned original two-stage approach, the estimation is performed based on a designed fixed grid, and the estimated values of the channel parameters are selected from the candidates that uniformly located on the grid. The accuracy of the estimated results all depends on the resolution of the grid. Hence, for high accuracy, the interval of the grid should be set as small as possible at the cost of more candidates on the grid. To obtain an accurate estimate of path delays or residual Doppler factors the grid needs to contain quite a number of candidates, leading to considerably high complexity of the estimation.

To obtain acceptable accuracy with lower complexity, we propose a novel channel estimation algorithm based on the two-stage strategy. The new method also uses the idea of greedy searching. In the first stage, unlike the aforementioned two-stage approach with OMP, the proposed algorithm estimates the path's initial delays by adaptively optimizing the delay grids during each iteration [6]. In the second stage, based on the estimatd path delays the residual Doppler factors are estimated in a similar way, and the corresponding path gains are obtained by the least squares method. The new method is summarized in Algorithm I as follows.

Algorithm I

Stage 1: Estimate the path delays $\{\tau_p\}_{p=1}^{N_\mathrm{p}}$ with the front training symbols $\mathbf{s}_\mathrm{f}$ and the corresponding received observation sequence $\mathbf{z}_\mathrm{f}$:

1) Parameter initialization: Initialize the delay grid $\mathcal{T}=\mathcal{T}^{(0)}=\{0,\tau_{\max}/\lambda,2\tau_{\max}/\lambda,...,\tau_{\max}\}$. Set the residual vector $\mathbf{r}=\mathbf{r}_0=\mathbf{z}_\mathrm{f}$, the set of the estimated values of initial path delays $\mathcal{L}_0=\emptyset$, and let $\hat{\mathbf{D}}_0$ be an empty matrix. Set the maximum iteration number Q for every path, and the maximum path number P. Set the threshold for the final residual power as η, and the threshold for the difference between the residual powers of two adjacent estimates of path delays as γ. Set the initial path number $p=1$, and the end flag $f=1$.

Loop 1: **while** $||\mathbf{r}||^2>\eta$ and $p\leq P$ and $f==1$

2) Estimate the initial delay of the pth path:

i. Perform the rough estimation of τ_p with the formula

$$\hat{\tau}_p^{(0)}=\arg\max_{\tau\in\mathcal{T}}|\langle\mathbf{d}_\tau,\mathbf{r}_{p-1}\rangle|,$$

where $\mathbf{d}_\tau=\mathbf{\Phi}_\tau^{(0)}\mathbf{W}_\tau\mathbf{s}_\mathrm{f}$;

ii. Set $q=1$;

Loop 2: **while** $q\leq Q$

iii. Set a new delay grid based on $\hat{\tau}_p^{(q-1)}$:

$$\mathcal{T}_p^{(q)}=\left\{\hat{\tau}_p^{(q-1)}-\frac{\tau_{\max}}{2^q\lambda},\hat{\tau}_p^{(q-1)},\hat{\tau}_p^{(q-1)}+\frac{\tau_{\max}}{2^q\lambda}\right\}$$

iv. Update the estimate of τ_p to make it finer with the formula

$$\hat{\tau}_p^{(q)}=\arg\max_{\tau\in\mathcal{T}_p^{(q)}}|\langle\mathbf{d}_\tau,\mathbf{r}_{p-1}\rangle|$$

v. Set $q=q+1$;

End *Loop 2*.

vi. Set the final estimate of the path delay: $\hat{\tau}_p=\hat{\tau}_p^{(Q)}$.

3) Solve the least squares problem:

$$\begin{aligned}\hat{\mathbf{a}}_p&=\arg\min_{\mathbf{a}}||\mathbf{z}_\mathrm{f}-\hat{\mathbf{D}}_p\mathbf{a}||_2\\&=(\hat{\mathbf{D}}_p^H\hat{\mathbf{D}}_p)^{-1}\hat{\mathbf{D}}_p^H\mathbf{z}_\mathrm{f}\end{aligned},$$

where $\hat{\mathbf{D}}_p=\left[\hat{\mathbf{D}}_{p-1}\ \mathbf{d}_{\hat{\tau}_p}\right]$.

4) Update the residual vector: $\mathbf{r}=\mathbf{r}_p=\mathbf{z}_\mathrm{f}-\hat{\mathbf{D}}_p\hat{\mathbf{a}}_p$.

5) Perform the following condition statement:

if $||\mathbf{r}_{p-1}||^2-||\mathbf{r}_p||^2>\gamma$,

set $\mathcal{L}=\mathcal{L}_p=\mathcal{L}_{p-1}\cup\{\hat{\tau}_p\}$, and $p=p+1$;

else

set $\mathcal{L}=\mathcal{L}_{p-1}$, and $f=0$;

end if

End *Loop 1*.

6) Obtain the total number of paths: $N_\mathrm{p}=p$.

Stage 2: Estimate the path residual Doppler factors $\{\beta_p\}_{p=1}^{N_\mathrm{p}}$ and the path gains $\{a_p\}_{p=1}^{N_\mathrm{p}}$ with the back training symbols $\mathbf{s}_\mathrm{b}$ and the corresponding received observation sequence $\mathbf{z}_\mathrm{b}$:

7) Parameter initialization: Initialize the Doppler grid $\mathcal{B}=\{-\beta_{\max},-\beta_{\max}+\frac{\beta_{\max}}{\xi},-\beta_{\max}+\frac{2\beta_{\max}}{\xi},...,\beta_{\max}\}$. Reset the residual vector $\mathbf{r}=\mathbf{r}_0=\mathbf{z}_\mathrm{b}$, and $\hat{\mathbf{D}}_0$ as an empty matrix. Set the maximum iteration number as Q.

Loop 3: **for** $p=1,2,\cdots,N_\mathrm{p}$

8) Estimate the residual Doppler factor of the pth path:

i. Perform the rough estimation of β_p with the formula

$$\hat{\beta}_p^{(0)}=\arg\max_{\beta\in\mathcal{B}}|\langle\mathbf{d}_{\hat{\tau}_p,\beta},\mathbf{r}_{p-1}\rangle|$$

where $\mathbf{d}_{\hat{\tau}_p,\beta}=\mathbf{\Phi}_{\hat{\tau}_p,\beta}^{(0)}\mathbf{W}_{\hat{\tau}_p,\beta}\mathbf{s}_\mathrm{b}$.

ii. Set $q=1$;

Loop 4: **while** $q\leq Q$

iii. Set a new Doppler grid based on $\hat{\beta}_p^{(q-1)}$:

$$\mathcal{B}_p^{(q)}=\left\{\hat{\beta}_p^{(q-1)}-\frac{\beta_{\max}}{2^q\xi},\hat{\beta}_p^{(q-1)},\hat{\beta}_p^{(q-1)}+\frac{\beta_{\max}}{2^q\xi}\right\}$$

iv. Update the estimate of β_p to make it finer with

$$\hat{\beta}_p^{(q)}=\arg\max_{\beta\in\mathcal{B}_p^{(q)}}|\langle\mathbf{d}_{\hat{\tau}_p,\beta},\mathbf{r}_{p-1}\rangle|$$

v. Set $q = q + 1$;

End *Loop 4*.

9) Solve the least squares problem:

$$\begin{aligned}\hat{\mathbf{a}}_p &= \arg\min_{\mathbf{a}} \|\mathbf{z}_{\rm b} - \hat{\mathbf{D}}_p \mathbf{a}\|_2 \\ &= (\hat{\mathbf{D}}_p^H \hat{\mathbf{D}}_p)^{-1} \hat{\mathbf{D}}_p^H \mathbf{z}_{\rm b}\end{aligned},$$

where $\hat{\mathbf{D}}_p = \left[\hat{\mathbf{D}}_{p-1}\ \mathbf{d}_{\hat{\tau}_p, \hat{\beta}_p}\right]$.

10) Update the residual vector: $\mathbf{r} = \mathbf{r}_p = \mathbf{z}_{\rm b} - \hat{\mathbf{D}}_p \hat{\mathbf{a}}_p$.

end *Loop 3*

11) Obtain the estimate of all the path gains: Set the estimated vector of path gains $\hat{\mathbf{a}} = \hat{\mathbf{a}}_{N_{\rm p}}$. The final estimate of the pth path gain is $\hat{a}_p = \hat{\mathbf{a}}[p]$, $p = 1, \cdots, N_{\rm p}$.

In either stage of Algorithm I, we first design a rough parameter grid, on which the elements are equally distributed. Based on the grid we start the iterations to search each path. In each iteration, a new grid with three candidates is designed based on the result of the last iteration, and the interval is reduced to half of the previous one, thus narrowing the search and increasing the resolution. By greedy searching over the updated grid, a new estimate is obtained. In this way, the estimate approaches the true value as the iterations progress, and the total candidates of all the grids are much less than those in the original two-stage approach with OMP.

C. Path Tracking Method

Since the duration of a frame is quite short, the changes in both path numbers and path delays between two adjacent frames are very small [7], [8]. Therefore, based on the estimate of the delays within the previous frame, we can construct some new searching grids with fewer candidates to track the current channel.

Considering the mth frame of the received signal, given the estimated path delays of the last frame: $\hat{\tau}_p[m-1]$, $p = 1, \cdots, N_{\rm p}[m-1]$, where $N_{\rm p}[m-1]$ is the estimated path number within the previous frame. The tracking method is summarized in Algorithm II as follows.

Algorithm II

Stage 1: Track the path delays $\{\tau_p\}_{p=1}^{N_{\rm p}[m]}$ with the front training symbols $\mathbf{s}_{\rm f}$ and the corresponding received observation sequence $\mathbf{z}_{\rm f}$:

1) Parameter initialization: Set the maximum iteration number Q for every path, and the maximum path number P. Initialize the path delay grid as

$$\mathcal{T} = \mathcal{T}^{(0)} = \{\mathcal{T}_1^{(0)}, \cdots, \mathcal{T}_{N_{\rm p}[m-1]}^{(0)}\},$$

in which

$$\mathcal{T}_p^{(0)} = \left\{\hat{\tau}_p[m-1] - \frac{\tau_{\max}}{\lambda}, \hat{\tau}_p[m-1], \hat{\tau}_p[m-1] + \frac{\tau_{\max}}{\lambda}\right\}$$

λ is defined as the initial resolution factor of the estimation of the delay. Set the residual vector $\mathbf{r} = \mathbf{r}_0 = \mathbf{z}_{\rm f}$, the set of the estimated values of initial path delays $\mathcal{L}_0 = \emptyset$, and let $\hat{\mathbf{D}}_0$ be an empty matrix. Set the threshold for the final residual power as η, and the threshold for the difference between the residual powers of two adjacent estimates of path delays as γ. Set the initial path number $p = 1$, and the end flag $f = 1$.

Loop 1: **while** $p \le N_{\rm p}[m-1]$ and $\|\mathbf{r}\|^2 > \eta$ and $f == 1$

2) Update estimate of the delay of the pth path: Perform the same operations as step 2) in Algorithm I to obtain the updated estimate of the initial path delay $\hat{\tau}_p$.

3) The same as step 3) in Algorithm I.

4) The same as step 4) in Algorithm I.

5) Abandon the used candidates of the delay grid: Set $\mathcal{T} = \mathcal{T} - \mathcal{T}_p^{(0)}$.

6) Perform the following condition statement:

if $\|\mathbf{r}_{p-1}\|^2 - \|\mathbf{r}_p\|^2 > \gamma$,

set $\mathcal{L} = \mathcal{L}_p = \mathcal{L}_{p-1} \cup \{\hat{\tau}_p\}$, and $p = p + 1$;

else

set $\mathcal{L} = \mathcal{L}_{p-1}$, $p = p - 1$,and $f = 0$;

end if

End *Loop 1*.

7) Estimate the new paths' initial delays:

Reset $f = 1$;

Loop 2: **while** $\|\mathbf{r}\|^2 > \eta$ and $p \le P$ and $f == 1$

Perform the same operations as steps *2)* to *5)* in Algorithm I;

end *Loop 2*.

8) Obtain the total number of paths with the current frame: $N_{\rm p}[m] = p$.

Stage 2: Estimate the residual Doppler factors $\{\beta_p\}_{p=1}^{N_{\rm p}[m]}$ and the path gains $\{a_p\}_{p=1}^{N_{\rm p}[m]}$ with the back training symbols $\mathbf{s}_{\rm b}$ and the corresponding received observation sequence $\mathbf{z}_{\rm b}$

9) Perform the same operations as stage 2 of Algorithm I: Obtain the estimate of path residual Doppler factors $\hat{\beta}_1, \hat{\beta}_2, \cdots, \hat{\beta}_{N_{\rm p}[m]}$ and the estimate of path gains $\hat{a}_1, \hat{a}_2, \cdots, \hat{a}_{N_{\rm p}[m]}$.

In Algorithm II, first we design a initial delay grid based on the estimate of the delays within the previous frame. Then we update the estimate of each path delay with the same adaptive grid optimization technique in Algorithm I, and determine whether the path survives or not according to the decrement of the residual vector's power. If the decrement is small enough, we discard the path and search for new paths. The process of tracking the residual Doppler factors and path gains is the same as that of stage 2 in Algorithm I.

D. Analysis of Complexity

For the compressed sensing algorithms with greedy searching, like OMP, the computational complexity is dominated by the dictionary's size [9] at each iteration. Let S

denote the length of training sequence, and K the number of candidates on the searching grid, then the complexity of an iteration is $\mathcal{O}(SK)$. Since S is fixed for estimation, so the complexity of a channel estimation/tracking method is proportional to K. Thus the number of all candidates on the grids constructed during the estimtion can be regarded as a measure of complexity. We use C to denote it for convenience.

Now we compare the computational complexities of the one-stage method with OMP, the two-stage method with OMP, the proposed estimation algorithm and the proposed tracking algorithm. Given: the channel to be estimated contains N_p paths; the final resolution of the estimate of delays is $\tau_{\max}/\left(2^Q\lambda\right)$, and the final resolution of the estimate of Doppler factors is $\beta_{\max}/(2^Q\xi)$. Thus, we can figure out the theoretical complexity of the one-stage method with OMP

$$C_{\mathrm{OMP1}} = [(2^Q\lambda+1)(2^{Q+1}\xi+1)-N_\mathrm{p}+1]N_\mathrm{p}/2, \quad (18)$$

the theoretical complexity of the two-stage method with OMP

$$C_{\mathrm{OMP2}} = (2^Q\lambda+2^{Q+1}\xi N_\mathrm{p}+2)N_\mathrm{p}, \quad (19)$$

the theoretical complexity of the proposed estimation method

$$C_{\mathrm{AlgI}} = (\lambda+4Q+2\xi+2)N_\mathrm{p}, \quad (20)$$

and the theoretical complexity of the proposed estimation method can be represented as

$$C_{\mathrm{AlgII}} = (N_\mathrm{p}+8Q+4\xi+5)N_\mathrm{p}/2. \quad (21)$$

The theoretical computational complexities of those four methods with the parameter Q are compared in Fig. 1. If the other parameters are fixed, the estimation accuracy will be determined by Q. The larger Q is, the more accurate the estimate will be. We can see clearly from the figure that when $Q>1$, the complexities of the proposed algorithms are much lower than those of the other two algorithms, and grow much slower as Q increases.

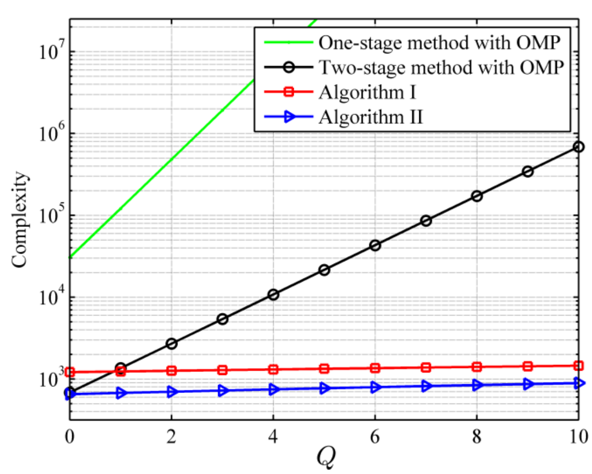

Fig. 1. The curves of four algorithms' complexities in relation to Q.

IV. Simulation Results

Considering such a doubly spread shallow water UWA channel with the depth 100m, the average sound speed 1500 m/s and the initial communication distance 2 km. Suppose the transmitter is moving towards the receiver at a horizontal speed 5m/s, which is quite large for UWA application, leading to serious Doppler spread. We use the Bellhop model [10] to calculate the channel state information including path delays, path Doppler scales, path attenuations, etc. 4 paths are selected for simulaiton for simplicty. The basic channel information is shown in TABLE I.

TABLE I. Basic Channel Information

Path Number	1	2	3	4
Delay/s	1.33375	1.33415	1.34371	1.34543
Gain	3.4195e-4	2.0825e-4	0.9848e-4	0.5266e-4
Doppler Scale	3.343445e-3	3.342417e-3	3.318691e-3	3.314377e-3

A. Comparison of Performance

A quadrature phase-shift keying (QPSK) signal with symbol rate f_b =2 kbaud and carrier frequency f_c =4 kHz is employed for transmission. Each frame contains 2500 symbols,.of which the first 150 are the front training symbols for estimating path delays and the last 300 are the back training symbols for estimating path residual Dopplers and gains. After resampling with the estimated mean Doppler scale and removing the carrier, the received signal in baseband can be used for estimating/tracking the channel.

First we initialize the parameters: set the maximum value of the estimate of path delays $\tau_{\max}$ = 0.02048 s, the maximum value of the estimate of path residual Dopplers $\beta_{\max}$ =5.12e-5, the maximum iteration number for each path $Q=3$, the resolution factor of the initial delay grid $\lambda=$256, and the resolution factor of the initial residual Doppler grid ξ =16. Then we use the original two-stage method with OMP and our proposed new estimation method to estimate the channel parameters $\left\{\{a_p^{(n)}\}_{n=0}^{N_\mathrm{a}}, \tau_p, \beta_p\right\}_{p=1}^{N_{\mathrm{pa}}}$ with the final delay resolution $\tau_{\max}/\left(2^Q\lambda\right)=$ 1e-5 s and Doppler resolution $\beta_{\max}/(2^Q\xi)$ =4e-7, respectively. The estimation results under the signal-to-noise ratio SNR=18 dB are shown in Fig. 2. From Fig. 2. we can see the estimaion errors of the two methods are both very small compared to the true channel information. It's hard to tell whose performance is better from the perspective of estimation accuracy.

Next we compare the recovery performance of the equalizers constructed by the estimates with different methods. With the estimated parameter set $\left\{\{\hat{a_p}^{(n)}\}_{n=0}^{N_\mathrm{a}}, \hat{\tau}_p, \hat{\beta}_p\right\}_{p=1}^{N_{\mathrm{pa}}}$, the channel matrix $\mathbf{H}$ can be construct using (12), (13) and (14). Once the channel matrix is obtained, the transmitted symbols can be recovered by linear minimum mean square error (MMSE) equalization [11]

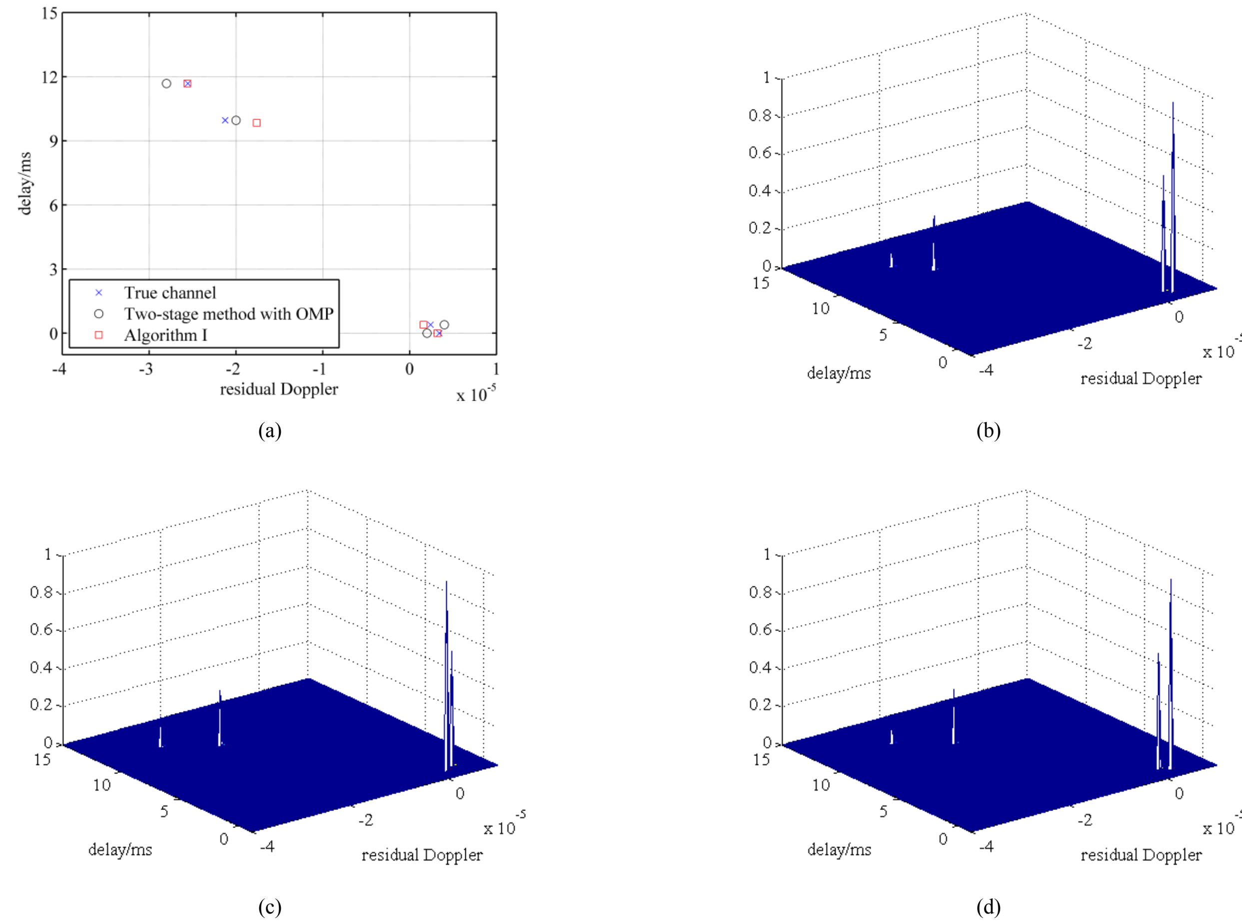

Fig. 2. (a) The distribution of the true channel parameters, the estimation results obtained by the exising two-stage OMP method.and the estimation results obtained by Algorithm I on the delay-Doppler plane. (b)The true channel parmeters. (c) The estimate obtained by the exising two-stage method with OMP. (d)The estimate obtained by the proposed estimation altoritm.

$$\hat{\mathbf{s}} = (\mathbf{H}^H\mathbf{H} + \sigma_{\mathbf{v}}^2\mathbf{I})^{-1}\mathbf{H}^H\mathbf{z}, \tag{22}$$

where $\sigma_{\mathbf{v}}^2$ denotes the noise variance, $\mathbf{I}$ denotes the identity matrix. With (13), we use the original two-stage OMP method, the proposed Algorithm I and Algorithm II to recover the transmitted sequences, respectively, and compare their symbol error rate (SER) performance and mean square error (MSE) performance. The three methods' SER curves and MSE curves are illustrated in Fig.3. and Fig.4. respectively. As is shown in both of the figures, all algorithms have good performances, which are quite close to the performance obtained with the true channel information. From the perspective of symbol recovery, the diffrence among the three algorithms' performances is negligible.

B. Comparison of Computational Complexity

Finally, we compare the computational complexity of the existing two-stage method with OMP, the proposed estimation algorithm and the proposed channel tracking algorithm. The theoretical complexities (the number of all candidates on the used searching grids during the estimation) of each algorithm can be calculated according to (19), (20) and (21) with the parameter we design. The theoretical complexities of the three algorithms are listed in TABLE II for reference. And the actual average running time of each algorithm in simulations is also listed there. Either from the perspective of theoretical complexity, or from the perspective of the running time, the proposed tracking algorithm has the lowest computational comlexity. The proposed estimation algorithm's complexity is a little higher than the proposed tracking algorithm's, but is far lower than the complexity of the existing two-stage method with OMP.

TABLE II. COMPLEXITY COMPARISON OF 3 ALGORITHMS

Term / Algorithm	Theoretical Complexity	Average Running Time in the Simulation/s
Two-stage OMP	12,296	7.500
Algorithm I	1,208	2.115
Algorithm II	776	1.232

In conclusion, according to the comparison of performance and the comparison of computational complexity, the proposed channel estimation and tracking method can achieve the same performance as the exixting two-stage OMP method with much lower complexity.

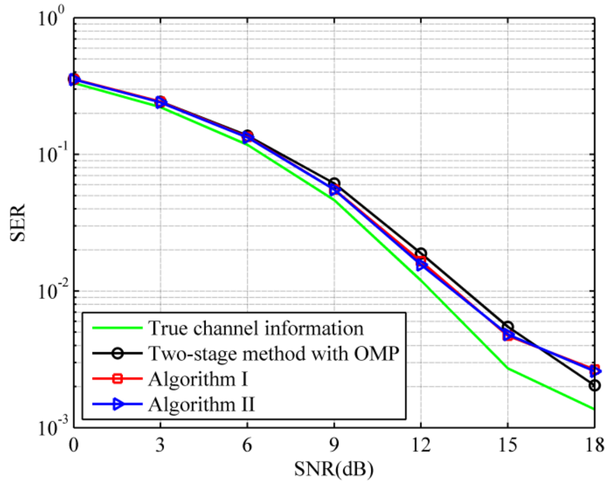

Fig. 3. SER performance comparison

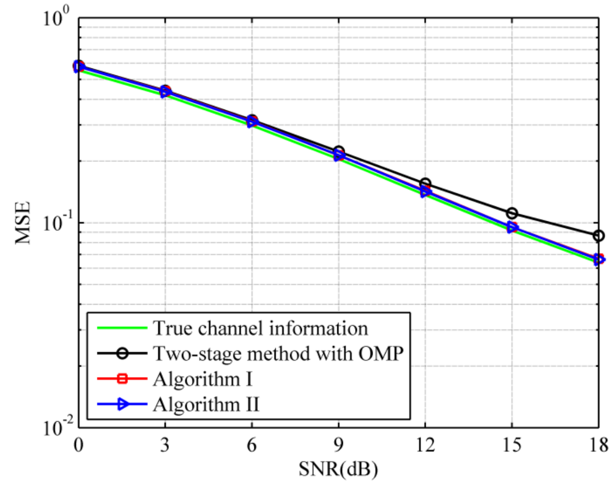

Fig. 4. MSE performance comparison

V. CONCLUSION

In this paper, a low-complexity method tailored for estimating and tracking sparse doubly spread channels in single-carrier coherent communications is proposed. In the new method, the strategy of two-stage estimation is adopted. In the first stage, the channel path delays are estimated iteratively via greedy searching over some adaptively optimized grids, which can dramatically reduce computational complexity. In the second stage, based on the estimated delays, the corresponding path residual Doppler factors are estimated iteratively with grid optimization as well, and the path gains are estimated one by one with the least squares method. According to the reasonable assumption that the path delays changes very little during a short period of time, we also propose a channel tracking algorithm. It takes advantage of the previous frame's esimation/tracking result, and updates each path's delay on the new optimized grids. In this way, the complexity is further reduced. Simulation results have demonstrated that the proposed method can achieve an identical performance with the existing two-stage method using OMP, but with far lower complexity.

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