# Sonar Detection Performance with LFM-BPSK Combined Waveforms

Li Jiang, Shefeng Yan, Yongqing Wu and Xiaochuan Ma Key Laboratory of Information Technology for Autonomous Underwater Vehicles, Chinese Academy of Sciences, Beijing, China, 100190 Email: sfyan@ieee.org

Abstract—As a sonar transmitted waveform, binary phaseshift keved (BPSK) signal has a nearly ideal ambiguity function. The good Doppler and time resolution properties make BPSK signal advantageous for underwater targets detection. Under the Doppler perturbation situation, the property of good doppler resolution will result in a sharp decline in detection performance. In recent years, waveform fusion has got extensive attentions. A novel waveform combining BPSK signal and a Doppler insensitive signal can improve the performance of target detection in case of perturbation. This article focuses on the study of ambiguity function, time and doppler resolution of LFM-BPSK signal, furthermore we studied the detection performance of linear frequency-modulated (LFM), BPSK and LFM-BPSK signals in the presence of perturbation. Simulation results show that the LFM-BPSK signal has a stronger anti-perturbation proformance than BPSK signal.

Index Terms—LFM-BPSK, sonar, detection performance, waveforms design, combined waveforms

### I. INTRODUCTION

Waveform selection is the key to the performance of active sonar systems, due to the diverse and complementary characteristics of transmitted signals [1]. An active sonar signal aims to achieve good range resolution and Doppler resolution has to have a large bandwidth and large time duration respectively. Among a great quantity of waveforms, a binary phase-shift keyed (BPSK) signal has good delay and Doppler resolution simultaneously. There is no interaction between this two parameter of a BPSK signal like with other types of signals, such as linear frequency-modulated (LFM) signal [2]. BPSK's Doppler resolution is the same as that of a continuous-wave (CW) signal of the same length, and its full range resolution is batter than that of an linear frequency-modulated waveform of the same bandwidth [3].

It may appear that the BPSK is the ideal sonar pulse and since it has existed already for quite some time, the question is raised why it is not popular in sonar applications. One of the most evident drawbacks is that the BPSK waveform is very Doppler sensitive. This implies that its performance is likely to be affected by Doppler perturbation [4]. There are various causes for Doppler perturbation such as reverberation influence or propagation through a time-varying medium or maneuvering extended target. For example, the range between source and target varies due to the helicopter's swing for an airborne sonar, which translates into a distortion of the Doppler spectrum of a given echo. The nature of this distortion leads

to spreading of the echo's Doppler spectrum which can case a notably decline of match filtering result.

Several scientists had studied on LFM-CW fusion waveform years ago [1], [5]. For a given time duration and a given bandwidth, CW signal corresponds to good Doppler but poor delay resolution capability; and that linear frequency-modulated pulses have the opposite behavior. The idea of fused LFM-CW signal is to achieve both the benefit of both signals. So one way to weaken this limitation may be fusing the BPSK signal with a Doppler insensitive signal, such as LFM signal, while maintaining a good delay and a appropriate Doppler resolution. The aim of this research is to investigate whether a combined LFM-BPSK can perform better than a pure BPSK signal under the Doppler perturbation.

The remainder of this paper is organized as follows. Section II describes the BPSK signal and its ambiguity function. In Section III, describes the proposed LFM-BPSK signal and analyzes its characteristic. Section IV evaluates the detection performance of the LFM-BPSK signal under the Doppler perturbation and compares its performance to that of BPSK signal and LFM signal and presents a discussion of the results. Finally, the conclusions are drawn in Section V.

### II. BPSK SIGNAL

The characteristics and performance of BPSK pulses for sonar detection have been developed by Jourdain in her founding work on semirandom pulses and their ambiguity surfaces [6], [7]. BPSK signal have been used for a variety of sonar applications [8]–[13].

A BPSK waveform is constructed by modulating the phase of a sinusoidal carrier of frequency transmitted for a duration. A pseudorandom binary sequence of bits of duration with good autocorrelation properties such as a maximum length pseudonoise sequence (m-sequence) is chosen [6]. Every bit change is coded by applying a phase jump of to the carrier. The signal u(t) with rectangular amplitude shading and pulse length T is an analytic signal expressed as

$$u(t) = c(t)e^{j2\pi f_0 t} \tag{1}$$

where  $f_0$  is the carrier frequency, c(t) is a binary m-sequence of N bits of each bit's duration  $\Delta = T/N$ 

$$c(t) = \frac{1}{\sqrt{T}} \sum_{k=0}^{N-1} q_k rect \left[ \frac{t - (k+1/2)\Delta}{\Delta} \right]$$
 (2)

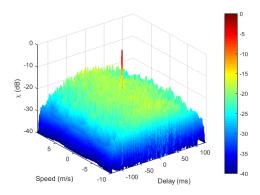


Fig. 1. BPSK signal's ambiguity function surface

where rect represents rectangular amplitude shading function, and  $q_k = 1$  or -1. Under noise-limited detection conditions, detection performance can be improved by increasing the signal energy. Under reverberation limited conditions, the reverberation level also increases with signal energy, performance is dependent on both signal duration and bandwidth, normally requiring wideband signals. The basic tool for performance characterization is the ambiguity function originally used to analyze a transmitted waveform. Assuming the target velocity is much lower than the speed of the medium and that the waveforms fractional bandwidth,  $B/2f_0$  is very low (i.e.1/10) which means that the signal can be well approximated as narrowband. In this case the ambiguity function can be further simplified to a narrowband model as

$$\chi(\tau,\varphi) = \int u(t)u^*(t+\tau)e^{j2\pi\varphi t}dt \tag{3}$$

where  $\varphi$  is the Doppler shift,  $\varphi = 2v/c$ . For the numerical examples, a BPSK pulse with a center frequency 25 kHz, a duration 128 ms, the length of m-sequence is 256 (it implies that the bandwidth is 2kHz). A rectangular window is used. This BPSK's narrowband ambiguity function shows in Fig. 1. It shows BPSK's ambiguity function has a high peak centered at zero delay and Doppler, with low sidelobe levels in both delay and Doppler. The sidelobe level (SLL) is relatively constant, without outliers, and its average level is [6]

$$SLL = -10log_{10}(N) \tag{4}$$

So the theoretically SLL of this BPSK signal is  $10log(256) \sim 24dB$ , the low sidelobe levels in both delay and Doppler are due to the cyclic orthogonality of the m-sequence. BPSK's ambiguity function also shows that there is no coupling between delay and Doppler as mentioned in Sec.I.

The full -3dB Doppler resolution in meters per second of such a pulse is [6]

$$\delta_{\nu} \approx \frac{c}{f_0 T} \tag{5}$$

where c is the sound speed in water. The full -3dB section of ambiguity function of aforementioned signal is shown in Fig. 2 (red line).

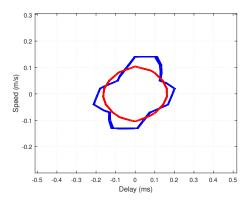


Fig. 2. -3dB section of LFM-BPSK (blue line) and BPSK (red line) Signal's Ambiguity Function

Fig. 2 demonstrates the doppler resolution happens to be the same as that of a CW waveform of the same length [14], it means the BPSK signal is a Doppler sensitive signal. In the next section, we will introduce the proposed LFM-BPSK combined waveform.

### III. THE LFM-BPSK SIGNAL

As we saw in Section II, the BPSK waveform is very Doppler sensitive. This implies that its match filtering result is likely to be affected by Doppler spreading. Consider Doppler spreading due to the movement between the source and the receiver, this Doppler spreading is called Doppler perturbation [4]. Inspired by Sun and Rago's works [1], [5], we aim to design a novel combined waveform. Precisely, this signal is constructed by sequentially combining BPSK and a non-sensitive signal(e.g. LFM signal).

The LFM signal is a widely used sonar transmitted waveform. The ambiguity function of a LFM signal as a analytical expression is

$$\chi(\tau,\varphi) = \left(1 - \frac{|\tau|}{T}\right) \ sinc[\pi(\varphi - M\tau)(T - |\tau|)] \tag{6}$$

where M is the modulation index of LFM signal. Consider a LFM signal with a center frequency 25 kHz, a duration 128 ms and a bandwidth 2kHz, its ambiguity function is shown in Fig. 3 As seen in Fig. 3 the peak values of the ambiguity surface stay nearly flat across Doppler axes, it is known to be insensitive to Doppler. So sequentially combining BPSK and LFM signal (LFM-BPSK) may decrease the influence from Doppler perturbation to the transmitted sonar signal. The LFM-BPSK signal comprises T/2 seconds of LFM followed by T/2 seconds of BPSK signal (which has the same bandwidth as the LFM signal). That is, we have

$$u(t) = \begin{cases} \frac{1}{2\sqrt{T}} rect\left(\frac{t}{2T}\right) e^{j2\pi(f_0t + \frac{1}{2}Mt^2)} & 0 \le t < T/2\\ \frac{1}{2\sqrt{T}} c(t) e^{j2\pi f_0t} & T/2 \le t < T\\ 0 & else \end{cases}$$
 (7)

The ambiguity function of this LFM-BPSK signal is shown in Fig. 4. From the Fig. 4 we can see that the ambiguity surface

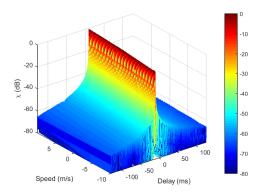


Fig. 3. LFM signal's ambiguity function surface

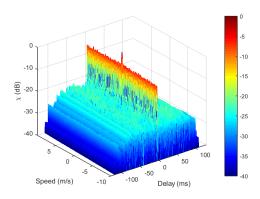


Fig. 4. LFM-BPSK signal's ambiguity function surface

of LFM-BPSK is also a kind of fusion of ambiguity surfaces of LFM and BPSK. One finds that the ambiguity surface has a peak at the original point of Doppler delay plane as same as BPSK's ambiguity function, Fig. 2 (blue line) shows the full -3dB of the LFM-BPSK ambiguity, one finds that the LFM-BPSK's Doppler and time resolution is slightly larger than the BPSK signal with same bandwidth and duration, it means the LFM-BPSK is a Doppler sensitive waveform and maintain the good Doppler and time resolution. However, due to the LFM part of LFM-BPSK signal, the sidelobe of this ambiguity function is relatively high, the sidelobe is nearly -6dB below the main peak. The high sidelobes could lead to an ambiguous or incorrect estimate of target Doppler, when there are multipaths returns or with strong reverberation. For example, the ambiguity surface of two returns, shifted in Doppler, will have two peaks in Doppler.

Fig. 5 shows the zero Doppler cut of ambiguity function (time response) of the three waveforms, one finds that BPSK and LFM-BPSK time response has much higher sidelobe than LFM, this is because the ambiguity volume constant principle. It can be seen that the LFM-BPSK time response has several sidelobe higher than the BPSK signal, this is because the LFM part of this fused signal will affect the cyclic orthogonality of the whole signal. Fig. 6 shows the zero Delay cut of ambiguity function (Doppler response) of the three waveforms, due to

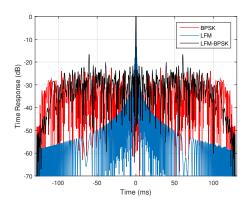


Fig. 5. Zero Doppler cut of ambiguity function.

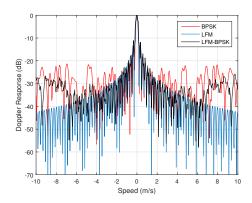


Fig. 6. Zero Delay cut of ambiguity function.

the Doppler insensitive features of LFM signal, the sidelobe of the LFM-BPSK Doppler response is higher than LFM but lower than the BPSK, it implies that the LFM-BPSK signal is a compromise of LFM and BPSK in Doppler detection.

## IV. DETECTION PERFORMANCE UNDER PERTURBATION

Consider a sonar mounted on a unmanned underwater vehicle, due to the inner wave or shift speed, it is hard to maintain a expected constant speed, this may cause a distortion of the Doppler spectrum of a transmitted waveform. We assume that the range is affected by a sinusoidal perturbation (corresponding to the pendular movement of the source) such that a given signal contains both positive and negative Doppler shifts. The period of this sinusoidal perturbation (128ms) is chosen equal to the signal duration. The expression of perturbation for a given transmitted waveform is

$$u_a(t) = u\left(t + a\sin 2\pi \frac{t}{T}\left(1 - \cos 2\pi \frac{t}{T}\right)\right) \tag{8}$$

where a is the amplitude of the distortion, and  $u_a(t)$  is the distorted pulse, T is the pulse length. To simulate this effect, we interpolated a 128ms on a distorted time vector with a = 0.0025ms shown in Fig. 7.

However an echo whose Doppler scale does not match with the match filters Doppler scale results in an SNR loss at the

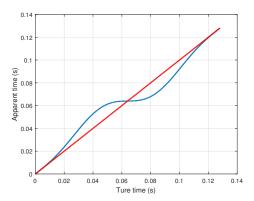


Fig. 7. Distorted time vector.

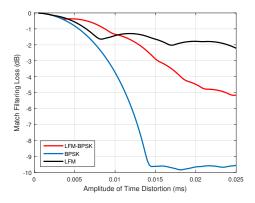


Fig. 8. Match filtering loss caused by time distortion of LFM-BPSK, LFM and BPSK signals.

output of the match filter. A loss in output SNR results in a reduction in detection performance. The amount of SNR loss depends upon the transmit waveform and how it responds to the Doppler Effect. Target velocity estimation is implemented with a bank of match filters with each match filter being tuned to a particular Doppler scale factor. The match filter that is the best match to the Doppler scaled echo will generate the strongest correlation to the echo. As a result this best matched match filter response will have the largest output. The Doppler scaling factor for that match filter is then taken as the estimate of the targets Doppler scaling factor and therefore velocity. We applied the distortion shown in Fig. 7 and a matched filter to LFM-BPSK, BPSK and BPSK signals of the same duration and bandwidth and plotted the match filter output SNR loss in Fig. 8. From the Fig. 8, one finds that BPSK signal match filter loss steeply descend with the increasing amplitude of time distortion, it reaches the lower bound (-10dB) at 0.015ms, meanwhile the match filter losses of LFM and LFM-BPSK are -1.5dB and -3dB respectively. With the increasing amplitude of time distortion, match filter losses of LFM and LFM-BPSK both descend, while both of the losses are smaller than the one of BPSK. The match filter loss of LFM-BPSK is mostly between the ones of LFM and BPSK.

### V. CONCLUSION

There are three sonar transmitted waveforms that have been discussed here i.e. LFM, BPSK, LFM-BPSK, each has unique characteristics that may be exploited to advantage in particular situations. The BPSK has good time and Doppler resolution, however, the detection performance will sharply decline under the perturbation situation. Meanwhile if one desires the good time and Doppler resolution like BPSK signal, LFM-BPSK waveform is a solution. The time and Doppler resolution of LFM-BPSK signal will decline compared with BPSK signal, but the match filter loss of LFM-BPSK doesn't decline as that. Thus it can be said LFM-BPSK waveform is a compromise solution. Future work is to be carried out on a lake trial, and bring even more insight on the detection performance analysis of LFM-BPSK under the influence of underwater channel.

#### ACKNOWLEDGMENT

This work was supported in part by the National Natural Science Foundation of China under Grant 61372181 and 61531018.

### REFERENCES

- [1] Y. Sun, P. Willett, and R. Lynch, "Waveform fusion in sonar signal processing," *Aerospace and Electronic Systems, IEEE Transactions on*, vol. 40, no. 2, pp. 462–477, 2004.
- [2] M. I. Skolnik, "Radar handbook," 1970.
- [3] N. Levanon, "Radar principles," New York, Wiley-Interscience, 1988, 320 p., vol. 1, 1988.
- [4] M. E. Colin and S. P. Beerens, "False-alarm reduction for low-frequency active sonar with bpsk pulses: experimental results," *Oceanic Engineer*ing, IEEE Journal of, vol. 36, no. 1, pp. 52–59, 2011.
- [5] C. Rago, P. Willett, and Y. Bar-Shalom, "Detection-tracking performance with combined waveforms," *Aerospace and Electronic Systems, IEEE Transactions on*, vol. 34, no. 2, pp. 612–624, 1998.
- [6] G. Jourdain and J. Henrioux, "Use of large bandwidth-duration binary phase shift keying signals in target delay doppler measurements," *The Journal of the Acoustical Society of America*, vol. 90, no. 1, pp. 299–309, 1991.
- [7] J. Millet and G. Jourdain, "Signaux à fort pouvoir de résolution tempsfréquence comparaison entre les signaux de costas et les signaux à modulation binaire de phase (bpsk)," *Traitement du signal*, vol. 7, no. 1, pp. 27–40, 1990.
- [8] T. Yang, J. Schindall, C.-F. Huang, and J.-Y. Liu, "Clutter reduction using doppler sonar in a harbor environment," *The Journal of the Acoustical Society of America*, vol. 132, no. 5, pp. 3053–3067, 2012.
- [9] K. Nakahira, S. Okuma, T. Kodama, and T. Furuhashi, "The use of binary coded frequency shift keyed signals for multiple user sonar ranging," in *Networking, Sensing and Control*, 2004 IEEE International Conference on, vol. 2. IEEE, 2004, pp. 1271–1275.
- [10] V. Carmillet and G. Jourdain, "Low-speed targets sonar detection using autoregressive models in reverberation; experimental performances for wideband signals," in *OCEANS'98 Conference Proceedings*, vol. 3. IEEE, 1998, pp. 1285–1289.
- [11] M. Van Gijzen and P. van Walree, "Shallow-water acoustic communication with high bit rate bpsk signals," in OCEANS 2000 MTS/IEEE Conference and Exhibition, vol. 3. IEEE, 2000, pp. 1621–1624.
- [12] B. K. Patel, D. Roy, and S. R. K. Vadali, "Performance of bpsk and bfsk digital modulation schemes in colored noise scenario," in *Underwater Technology (UT)*, 2015 IEEE. IEEE, 2015, pp. 1–5.
- [13] S. Wentao, H. Jianguo, C. Xiaodong, and H. Yunshan, "Orthogonal waveforms design and performance analysis for mimo sonar," in *Signal Processing (ICSP)*, 2010 IEEE 10th International Conference on. IEEE, 2010, pp. 2382–2385.
- [14] X. Lurton, An introduction to underwater acoustics: principles and applications. Springer Science & Business Media, 2002.