Symmetric spectrum detection in the presence of partially homogeneous environment

Chengpeng Hao*, Danilo Orlando†, Alfonso Farina‡, Salvatore Iommelli§ and Chaohuan Hou*
*Institute of Acoustics, Chinese Academy of Sciences, 100190 Beijing, China
†Università degli Studi "Niccolò Cusano", via Don Carlo Gnocchi 3, 00166 Roma, Italy
‡Selex-ES (retired), via Tiburtina Km. 12.4, 00131, Roma, Italy
§Ente di Formazione Professionale Maxwell, via G. A. Campano, 103/105, I-80145 Napoli, Italy
Email: haochengp@mail.ioa.ac.cn

***Abstract*—In this paper, we deal with the problem of detecting the signal of interest in the presence of Gaussian disturbance with symmetric spectrum and assuming that the cell under test (primary data) and the training samples (secondary data) share the same covariance matrix up to an unknown power scaling factor. Moreover, we exploit the symmetric spectral property of the disturbance to transfer the binary hypothesis testing problem from the complex to the real domain and derive an adaptive detector relying on the two-step Generalized Likelihood Ratio Test design procedure. A preliminary performance assessment, conducted by Monte Carlo simulation, has confirmed the effectiveness of the newly proposed detector compared with the traditional state-of-the-art counterpart which ignores the spectrum symmetry.**

I. Introduction

The problem of detecting a multichannel signal buried in temporally and spatially correlated disturbance (clutter plus noise) has been extensively studied in phased-array radar. In the past decades, a number of detection algorithms for point-like targets have been introduced to solve the problem. Most of them require the estimation of the space-time covariance matrix of the disturbance to suppress the interferences. Examples of such detectors include Kelly's Generalized Likelihood Ratio Test (GLRT) [1], the Adaptive Matched Filter (AMF) detector [2], the Adaptive Coherence Estimator (ACE) detector [3], and the Rao test [4], [5] etc. All of them suppose that a set of secondary data, free of signal components and sharing the same spectral properties of the primary data, is available to estimate the disturbance covariance matrix. However, realistic radar scenarios are often non-homogeneous because of environmental effects and system aspects, which drastically reduces the number of homogeneous secondary data and results in significant degradation in detection performance [6].

In order to circumvent the lack of a sufficient amount of homogenous secondary data, the knowledge-aided approach has recently gained significant attention. A natural way to incorporate prior knowledge in solving the detection problem is a Bayesian approach that models the disturbance covariance matrix as a random matrix with some prior [7]–[9]. These Bayesian detectors are modified versions of the standard AMF or GLRT through diagonal or colored-loading. Another efficient way to alleviate the requirement of the amount of training data is to exploit the persymmetric structural property of the covariance matrix. The proposed persymmetric detectors, obtained by accounting for the persymmetric property at the design stage, greatly improves the robustness in training-limited scenarios [10]–[12].

More recently, in [13], another source of a priori information, the symmetry in the clutter spectral characteristics which would reduce the number of nuisance parameters to estimate, is firstly exploited in the design of adaptive detection algorithms. This symmetry property implies that clutter autocorrelation function is real-valued and, hence, the original detection problem can be transferred from the complex domain to the real domain. Within this framework, the two-step GLRT design procedure [2] is exploited to devise an adaptive architecture for homogenous environment. This design procedure consists in evaluating the GLRT assuming that the clutter covariance matrix is known and maximizing over the other unknown parameters. Then, an appropriate estimate of the clutter covariance matrix based on the secondary data is substituted into this test.

In this work, we extend the framework proposed in [13] to take into account the partially homogeneous environment, where the primary data and the secondary data share the same covariance matrix up to an unknown power scaling factor. One motivation to consider the partially homogeneous model is due to the use of guard cells in radar signal processing, which may lead to a power difference between the primary data and the secondary data. At the design stage, we exploit the fact that the clutter spectrum is an even function and solve the new hypothesis test resorting to the two-step design procedure [2]. A preliminary performance analysis confirms the superiority of the considered architecture over its conventional counterpart which does not exploit the symmetric spectral property.

The remainder of this paper is organized as follows. Section II addresses the problem formulation, Section III deals with the design of the detector, and Section IV provides illustrative examples. Finally, Section V contains some concluding remarks.

A. Notation

In the sequel, vectors and matrices are denoted by boldface lower-case and upper-case letters, respectively. As to the numerical sets, $\mathbb{R}$ is the set of the real numbers, $\mathbb{R}^{N\times M}$

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is the set of the $(N \times M)$-dimensional real matrices, $\mathbb{C}$ is the set of the complex numbers, and $\mathbb{C}^{N\times M}$ is the set of the $(N \times M)$-dimensional complex matrices. The real and imaginary parts of a complex vector or scalar are denoted by $\Re(\cdot)$ and $\Im(\cdot)$, respectively. Symbols $(\cdot)^T$ and $(\cdot)^\dagger$ stand for transpose and conjugate transpose, respectively. Finally, the acronym iid means independent and identically distributed.

II. Problem Formulation

Assume that a sensing systems acquires data from $N \geqslant 2$ channels which can be spatial and/or temporal. The echoes from the cell under test are properly pre-processed; then, they are sampled and organized to form a N-dimensional vector, $\boldsymbol{r}$ say. We want to test whether or not $\boldsymbol{r}$ contains useful target echoes assuming the presence of a set of K secondary data. Summarizing, we can write this decision problem as follows

$$\begin{cases} H_0 : \begin{cases} \boldsymbol{r} = \boldsymbol{n}, \\ \boldsymbol{r}_k = \boldsymbol{n}_k, \quad k = 1, \ldots, K, \end{cases} \\ H_1 : \begin{cases} \boldsymbol{r} = \alpha \boldsymbol{v} + \boldsymbol{n}, \\ \boldsymbol{r}_k = \boldsymbol{n}_k, \quad k = 1, \ldots, K, \end{cases} \end{cases} \tag{1}$$

where

- $\boldsymbol{v} = \boldsymbol{v}_1 + j\boldsymbol{v}_2 \in \mathbb{C}^{N\times 1}$ with $\|\boldsymbol{v}\| = 1$, $\boldsymbol{v}_1 = \Re\{\boldsymbol{v}\}$, and $\boldsymbol{v}_2 = \Im\{\boldsymbol{v}\}$ is the nominal steering vector;
- $\alpha = \alpha_1 + j\alpha_2 \in \mathbb{C}$ with $\alpha_1 = \Re\{\alpha\}$ and $\alpha_2 = \Im\{\alpha\}$ represents the target response;
- $\boldsymbol{n} = \boldsymbol{n}_1 + j\boldsymbol{n}_2$ and $\boldsymbol{n}_k = \boldsymbol{n}_{1k} + j\boldsymbol{n}_{2k} \in \mathbb{C}^{N\times 1}$, $k = 1, \ldots, K$, with $\boldsymbol{n}_1 = \Re\{\boldsymbol{n}\}$, $\boldsymbol{n}_2 = \Im\{\boldsymbol{n}\}$, $\boldsymbol{n}_{1k} = \Re\{\boldsymbol{n}_k\}$, and $\boldsymbol{n}_{2k} = \Im\{\boldsymbol{n}_k\}$, are iid complex normal random vectors with zero mean and unknown positive definite covariance matrices given by

$$E\left[\boldsymbol{n}\boldsymbol{n}^\dagger\right] = \gamma \boldsymbol{M}_0, \quad E\left[\boldsymbol{n}_k\boldsymbol{n}_k^\dagger\right] = \boldsymbol{M}_0, \tag{2}$$

with $\gamma > 0$. Furthermore, we assume that the disturbance exhibits a power spectral density symmetric with respect to the zero frequency, which implies that $\boldsymbol{M}_0$ is real-valued, and $\boldsymbol{n}_1$, $\boldsymbol{n}_2$ and $\boldsymbol{n}_{1k}$, $\boldsymbol{n}_{2k}$, $k = 1, \ldots, K$, are iid Gaussian vectors with zero mean and covariance matrices

$$\begin{aligned} E\left[\boldsymbol{n}_1\boldsymbol{n}_1^\dagger\right] &= E\left[\boldsymbol{n}_2\boldsymbol{n}_2^\dagger\right] = \gamma \boldsymbol{M}, \\ E\left[\boldsymbol{n}_{1k}\boldsymbol{n}_{1k}^\dagger\right] &= E\left[\boldsymbol{n}_{2k}\boldsymbol{n}_{2k}^\dagger\right] = \boldsymbol{M}, \end{aligned} \tag{3}$$

with $\boldsymbol{M} = \frac{1}{2}\boldsymbol{M}_0 \in \mathbb{R}^{N\times N}$. Thus, problem (1) is equivalent to

$$\begin{cases} H_0 : \begin{cases} \boldsymbol{z}_1 = \boldsymbol{n}_1, \ \boldsymbol{z}_2 = \boldsymbol{n}_2, \\ \boldsymbol{z}_{1k} = \boldsymbol{n}_{1k}, \ \boldsymbol{z}_{2k} = \boldsymbol{n}_{2k}, \quad k = 1, \ldots, K, \end{cases} \\ H_1 : \begin{cases} \boldsymbol{z}_1 = (\alpha_1\boldsymbol{v}_1 - \alpha_2\boldsymbol{v}_2) + \boldsymbol{n}_1, \\ \boldsymbol{z}_2 = (\alpha_1\boldsymbol{v}_2 + \alpha_2\boldsymbol{v}_1) + \boldsymbol{n}_2, \\ \boldsymbol{z}_{1k} = \boldsymbol{n}_{1k}, \ \boldsymbol{z}_{2k} = \boldsymbol{n}_{2k}, \quad k = 1, \ldots, K. \end{cases} \end{cases} \tag{4}$$

Remark 1: Transferring problem (1) form complex domain to real domain, is equivalent to doubling the number of secondary data and, hence the new receiver obtained by solving problem (4) would work when $2K \geqslant N$ instead of $K \geqslant N$ which is required by the traditional detectors in [1]–[3]. Moreover, we expect that the new receiver exhibits superior detection performance with respect to its counterpart which ignores the spectrum symmetry.

III. Detector Design

In this section, we solve problem (4) resorting to the two-step GLRT design procedure which consists in evaluating the GLRT of the cell under test assuming that the covariance matrix $\boldsymbol{M}$ is known and then replacing it with a proper estimate. As a preliminary step toward the derivation of the receivers, let us denote by $\boldsymbol{Z} = [\boldsymbol{z}_1 \ \boldsymbol{z}_2]$ the primary data matrix and $\boldsymbol{Z}_S = [\boldsymbol{z}_{11} \ \ldots \ \boldsymbol{z}_{1K} \ \boldsymbol{z}_{21} \ \ldots \ \boldsymbol{z}_{2K}]$ the secondary data matrix. Under the assumption that $\boldsymbol{M}$ is known, the GLRT is given by [14]

$$\frac{\max\limits_{\alpha_1,\alpha_2} \max\limits_{\gamma} f_1(\boldsymbol{Z}; \boldsymbol{M}, \gamma, \alpha_1, \alpha_2)}{\max\limits_{\gamma} f_0(\boldsymbol{Z}; \boldsymbol{M}, \gamma)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \tag{5}$$

where η is the threshold value to be set according to the desired Probability of False Alarm (P_{fa}), and $f_j(\boldsymbol{Z}, \cdot)$ is the probability density functions (PDF) of primary data under H_j, $j = 0, 1$, namely

$$\begin{aligned} f_0(\boldsymbol{Z}; \boldsymbol{M}, \gamma) &= \frac{1}{(2\pi)^N \det(\gamma \boldsymbol{M})} \\ &\times \exp\left\{-\frac{1}{2}\mathrm{Tr}\left[\frac{1}{\gamma}\boldsymbol{M}^{-1}\boldsymbol{Z}\boldsymbol{Z}^T\right]\right\}, \end{aligned} \tag{6}$$

and

$$\begin{aligned} f_1(\boldsymbol{Z}; \boldsymbol{M}, \gamma, \alpha_1, \alpha_2) &= \frac{1}{(2\pi)^N \det(\gamma \boldsymbol{M})} \\ &\times \exp\left\{-\frac{1}{2}\mathrm{Tr}\left[\frac{1}{\gamma}\boldsymbol{M}^{-1}\left(\boldsymbol{u}_1\boldsymbol{u}_1^T + \boldsymbol{u}_2\boldsymbol{u}_2^T\right)\right]\right\}, \end{aligned} \tag{7}$$

where $\boldsymbol{u}_1 = \boldsymbol{z}_1 - \alpha_1\boldsymbol{v}_1 + \alpha_2\boldsymbol{v}_2$, and $\boldsymbol{u}_2 = \boldsymbol{z}_2 - \alpha_1\boldsymbol{v}_2 - \alpha_2\boldsymbol{v}_1$.

It is easy to show that the MLEs of γ under H_0 and H_1 are given by

$$\begin{aligned} \widehat{\gamma}_0 &= \frac{\mathrm{Tr}\left[\boldsymbol{M}^{-1}\boldsymbol{Z}\boldsymbol{Z}^T\right]}{2N}, \\ \widehat{\gamma}_r &= \frac{\mathrm{Tr}\left[\boldsymbol{M}^{-1}\left(\boldsymbol{u}_1\boldsymbol{u}_1^T + \boldsymbol{u}_2\boldsymbol{u}_2^T\right)\right]}{2N}. \end{aligned} \tag{8}$$

Substituting (7) and (8) in (5), after some algebraic manipulations, the natural logarithm of (5) can be recast as

$$\frac{\boldsymbol{z}_1^T\boldsymbol{M}^{-1}\boldsymbol{z}_1 + \boldsymbol{z}_2^T\boldsymbol{M}^{-1}\boldsymbol{z}_2}{\min\limits_{\alpha_1,\alpha_2} f(\alpha_1, \alpha_2)} \underset{H_0}{\gtrless} \eta, \tag{9}$$

where η is the suitable modification of the threshold in (5), and

$$f(\alpha_1, \alpha_2) = \boldsymbol{u}_1^T\boldsymbol{M}^{-1}\boldsymbol{u}_1 + \boldsymbol{u}_2^T\boldsymbol{M}^{-1}\boldsymbol{u}_2. \tag{10}$$

In the next step, our objective is to minimize $f(\alpha_1, \alpha_2)$ with respect to α_1 and α_2. To this end, we evaluate the first derivatives with respect to α_1 and α_2, which are given by

$$\begin{aligned} \frac{\partial f(\alpha_1, \alpha_2)}{\partial \alpha_1} &= -2\boldsymbol{v}_1^T\boldsymbol{M}^{-1}\boldsymbol{u}_1 - 2\boldsymbol{v}_2^T\boldsymbol{M}^{-1}\boldsymbol{u}_2, \\ \frac{\partial f(\alpha_1, \alpha_2)}{\partial \alpha_2} &= 2\boldsymbol{v}_2^T\boldsymbol{M}^{-1}\boldsymbol{u}_1 - 2\boldsymbol{v}_1^T\boldsymbol{M}^{-1}\boldsymbol{u}_2. \end{aligned} \tag{11}$$

Setting to zero the two derivatives of (11), yields

$$\widehat{\alpha}_r = \frac{\boldsymbol{v}_1^T \boldsymbol{M}^{-1} \boldsymbol{z}_1 + \boldsymbol{v}_2^T \boldsymbol{M}^{-1} \boldsymbol{z}_2}{\boldsymbol{v}_1^T \boldsymbol{M}^{-1} \boldsymbol{v}_1 + \boldsymbol{v}_2^T \boldsymbol{M}^{-1} \boldsymbol{v}_2},$$
$$\widehat{\alpha}_i = \frac{\boldsymbol{v}_1^T \boldsymbol{M}^{-1} \boldsymbol{z}_2 - \boldsymbol{v}_2^T \boldsymbol{M}^{-1} \boldsymbol{z}_1}{\boldsymbol{v}_1^T \boldsymbol{M}^{-1} \boldsymbol{v}_1 + \boldsymbol{v}_2^T \boldsymbol{M}^{-1} \boldsymbol{v}_2}. \tag{12}$$

Based on the above results, the GLRT can be recast as

$$\frac{\left(\boldsymbol{v}_1^T \boldsymbol{M}^{-1} \boldsymbol{z}_1 + \boldsymbol{v}_2^T \boldsymbol{M}^{-1} \boldsymbol{z}_2\right)^2 + \left(\boldsymbol{v}_1^T \boldsymbol{M}^{-1} \boldsymbol{z}_2 - \boldsymbol{v}_2^T \boldsymbol{M}^{-1} \boldsymbol{z}_1\right)^2}{\left(\boldsymbol{z}_1^T \boldsymbol{M}^{-1} \boldsymbol{z}_1 + \boldsymbol{z}_2^T \boldsymbol{M}^{-1} \boldsymbol{z}_2\right)\left(\boldsymbol{v}_1^T \boldsymbol{M}^{-1} \boldsymbol{v}_1 + \boldsymbol{v}_2^T \boldsymbol{M}^{-1} \boldsymbol{v}_2\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{13}$$

The most natural estimator of $\boldsymbol{M}$ in Gaussian disturbance is the sample covariance matrix based on the secondary data, namely, $\boldsymbol{S} = \boldsymbol{Z}_S \boldsymbol{Z}_S^\dagger$. Plugging $\boldsymbol{S}$ in place of $\boldsymbol{M}$ into (13), the GLRT is finally given by

$$\frac{\left(\boldsymbol{v}_1^T \boldsymbol{S}^{-1} \boldsymbol{z}_1 + \boldsymbol{v}_2^T \boldsymbol{S}^{-1} \boldsymbol{z}_2\right)^2 + \left(\boldsymbol{v}_1^T \boldsymbol{S}^{-1} \boldsymbol{z}_2 - \boldsymbol{v}_2^T \boldsymbol{S}^{-1} \boldsymbol{z}_1\right)^2}{\left(\boldsymbol{z}_1^T \boldsymbol{S}^{-1} \boldsymbol{z}_1 + \boldsymbol{z}_2^T \boldsymbol{S}^{-1} \boldsymbol{z}_2\right)\left(\boldsymbol{v}_1^T \boldsymbol{S}^{-1} \boldsymbol{v}_1 + \boldsymbol{v}_2^T \boldsymbol{S}^{-1} \boldsymbol{v}_2\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{14}$$

Interestingly, it is easy to show that (14) can be expressed in terms of the original complex vectors $\boldsymbol{v}$ and $\boldsymbol{r}$ as follows

$$\frac{\left|\boldsymbol{v}^T \boldsymbol{S}^{-1} \boldsymbol{r}\right|^2}{\left(\boldsymbol{r}^T \boldsymbol{S}^{-1} \boldsymbol{r}\right)\left(\boldsymbol{v}^T \boldsymbol{S}^{-1} \boldsymbol{v}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \tag{15}$$

Apparently, detector (15) shares the same detection structure as the ACE, with the only difference being that the SCM based on $\boldsymbol{Z}_K$ is real and takes the place of the usual SCM based upon $\boldsymbol{r}_k$, $k = 1, \ldots, K$. Accordingly, detector (15) will be referred to in the sequel as the Symmetric Spectrum ACE (SS-ACE).

IV. Performance Assessment

This section is devoted to the performance assessment of the newly proposed detector in terms of Probability of Detection ($P_{\rm d}$). Moreover, the Constant False Alarm Rate (CFAR) property is investigated. To this end, we firstly examine the scale invariance property of the SS-ACE in comparison with the so-called Symmetric Spectrum AMF (SS-AMF) introduced in [13]. Secondly, we compare the new receiver with the ACE in PHE. In the examples, we also include the curves of the ACE for known $\boldsymbol{M}_0$, which cannot be used in practice but offers a baseline for comparison. This detector is referred to in the sequel as the benchmark detector.

Since the closed form expressions for the $P_{\rm d}$ and the $P_{\rm fa}$ are not available, we make use of standard Monte Carlo counting techniques and evaluate the thresholds necessary to ensure the preassigned value $P_{\rm fa} = 10^{-4}$ resorting to $100/P_{fa}$ independent trials. On the other hand, the P_d values are estimated over 10^4 independent trials. As to the disturbance model, we assume a clutter-dominated environment with the covariance matrix $\boldsymbol{M}_0 = \sigma_n^2 \boldsymbol{I}_N + \sigma_c^2 \boldsymbol{M}_c$, where σ_n^2 is the thermal noise power, σ_c^2 is the clutter power which is evaluated according to a pre-designed Clutter-to-Noise Ratio (CNR), defined as CNR $= \sigma_c^2/\sigma_n^2$. As to $\boldsymbol{M}_c$, it is Gaussian shaped with one-lag correlation coefficient ρ. Precisely, the (i,j)th element of $\boldsymbol{M}_c$ is $\rho^{|i-j|^2}$ with $\rho = 0.9$. The steering vector $\boldsymbol{v}$ is given by $\boldsymbol{v} = [1, \ldots, 1]^T/\sqrt{N}$, and the signal-to-noise ratio (SNR) is defined as SNR $= |\alpha|^2 \boldsymbol{v}^\dagger \boldsymbol{M}_0^{-1} \boldsymbol{v}$.

$$\text{SNR} = |\alpha|^2 \boldsymbol{v}^\dagger \boldsymbol{M}_0^{-1} \boldsymbol{v}. \tag{16}$$

A. CFAR analysis

We first examine the invariance of the SS-ACE and the SS-AMF with respect to the scaling factor γ. Precisely, via Monte Carlo simulations, we determine the threshold for each test corresponding to a given $P_{\rm fa}$ for $N = 16$, $K = 32$, CNR $=$ 60 dB, and γ varying from 1 to 51 in a step size of 10. For the convenience of computer simulation, we choose $P_{\rm fa} = 10^{-2}$. The results are shown in Fig. 1. As it can be seen, the SS-ACE have a constant threshold independent of the considered values of γ. In contrast, the SS-AMF are more sensitive to the variation of γ, because it give thresholds with two distinct phases: a linearly decreasing phase when γ is small; and a saturated phase when is large, e.g., greater than 10 in this example.

B. Performance of detection

In Fig. 2, we study the detection performance of the SS-ACE and the ACE assuming $N = 16$, $K = 17$, $P_{\rm fa} = 10^{-4}$, CNR $=$ 60 dB, and $\gamma = 3$. As it can be seen from Fig. 2, the SS-ACE guarantees a $P_{\rm d}$ gain with more than 15 dB with respect to the ACE. Thus, incorporating the *a priori* knowledge is a very effective means to improve performance in the presence of a small number of secondary data. We would like to point out that such an improvement is a theoretical value. In practice, it will be not fully realistic to have a perfectly symmetric doppler spectrum and, hence, the $P_{\rm d}$ gain will decrease accordingly. However the aforementioned gain reduces when a sufficient amount of secondary data is available. This is shown in Fig. 3, which assumes the same system parameters as in Fig. 2, but for $K = 32$. As expected, the gain of the SS-ACE over the ACE reduces to about 2 dB.

Finally, In Fig. 4 we plot $P_{\rm d}$ against SNR for the SS-ACE assuming $K < N$. In particular, we set $N = 16$ and two cases of K, i.e., $K = 9$ and $K = 14$. The plots show that the SS-ACE can work steadily in the case of $K < N$ and that the larger K, the better $P_{\rm d}$ the SS-ACE has.

V. Conclusions

In this paper, we have proposed a decision scheme for adaptive detection in Gaussian clutter with symmetric spectrum for partially homogeneous environment. In order to derive the new detector, we transfer the binary hypothesis test problem from complex domain to real domain and resort to the two-step GLRT-based design procedure. The performance assessment has demonstrated that the proposed receiver can significantly outperform its natural competitor which ignores the spectrum symmetry in a scenario where a very small number of secondary data is available.

Acknowledgements

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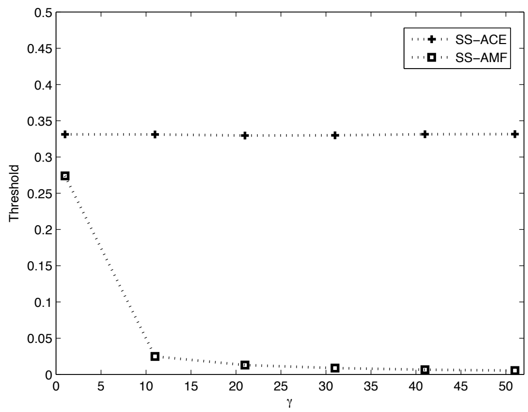

Fig. 1. Thresholds versus γ for the SS-ACE and the SS-AMF when $N = 16$, $K = 32$, $P_{\mathrm{fa}} = 10^{-2}$ and $\mathrm{CNR} = 60$ dB.

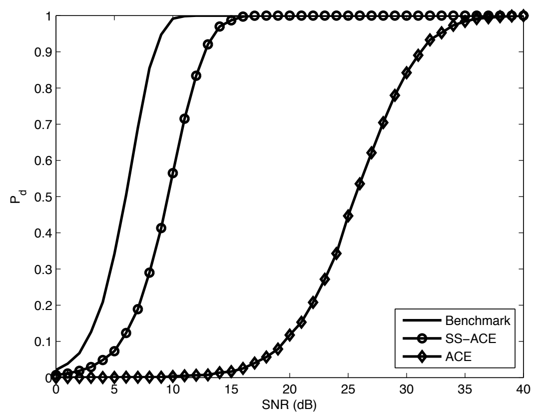

Fig. 2. P_{d} versus SNR for the SS-ACE and the ACE assuming $N = 16$, $K = 17$, $\gamma = 3$ and $\mathrm{CNR} = 60$ dB.

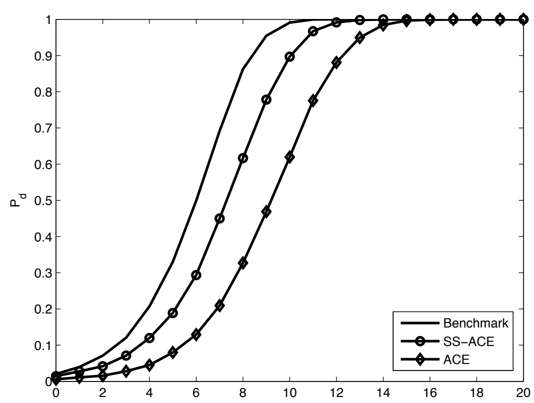

Fig. 3. P_{d} versus SNR for the SS-ACE and the ACE assuming $N = 16$, $K = 32$, $\gamma = 3$ and $\mathrm{CNR} = 60$ dB.

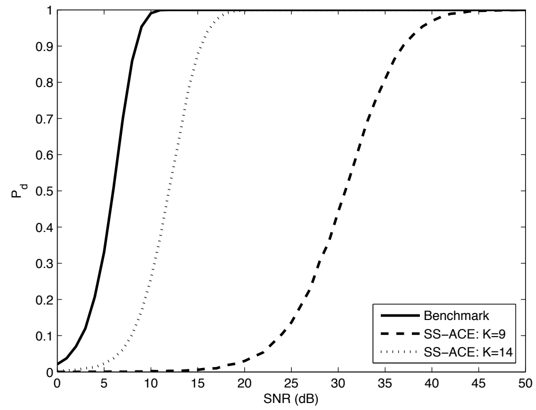

Fig. 4. P_{d} versus SNR for the SS-ACE assuming $N = 16$, $K < N$, $\gamma = 3$ and $\mathrm{CNR} = 60$ dB.

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