Measurements of Backward Wave Propagation Using the Dynamic Photoelastic Technique

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***Abstract*—Guided waves in plates and cylinders exhibit the backward propagation that group and phase velocities have opposite directions. Backward waves might be used to control the acoustic energy flux. The existence of backward waves in acoustic waveguides has been well established, but the direct visualization of the propagation of backward modes has not been widely reported. The dynamic photoelastic method is able to present the stress distribution inside a solid material. In this paper, the photoelasticity has been used to measure backward-wave motions in plates, in order to obtain a better understanding of propagation characteristics of backward waves.**

Keywords— backward wave; dynamic photoelastic technique; guided wave;

I. INTRODUCTION

Propagation characteristics of Lamb waves in a thin plate are typically represented by a set of dispersion curve [1-3] which are described by the Rayleigh-Lamb dispersion equation. Lamb modes exhibit backward-propagating branches which group and phase velocities have opposite directions. Backward waves have received intense interests due to the fact that it might provide new ways to manipulate acoustic fields [2]. This non-intuitive physical effects can be applied for the non-destructive characterization of materials or the Lamb-wave lens with a high numerical aperture [2-5]. All propagating modes of Lamb waves can exhibit backward wave propagations over some ranges of Poisson's ratio, except of the A0, A1, and S0 modes [2,3].

In 1904, Lamb discussed the possibility of the existence of the backward wave theoretically [1]. In 1957, Tolstoy and Usdin [6] found the existence of the backward mode S_{2b} in a free isotropic plate with the Poisson ' s ratio of 0.25, by numerically analyzing the characteristic equation. Experiments also performed. Meitzler [7] and Wolf [8] reported experimental observations of stress pulses travelling in backward elastic-wave motions in rods, which excited and detected by piezoelectric transducers made of ceramic materials. Negishi and Li [9] reported the observation of the A_{3b} mode in a BK7 glass plate detected by the photoelastic method. Prada [10] also excited the backward wave motions in plates and pipes using the laser ultrasonic method and detected the backward waves by the laser interferometer.

The existence of backward waves in acoustic waveguides is well established [2-4,10-11], but the direct visualization of the propagation of backward modes in plates has not been widely reported. Currently, the common method to observe backward wave is to use the laser interferometer [2,3,10], but the detection area is limited to the surface of a waveguide. The dynamic photoelastic method is able to study the stress distributions inside a transparent solid materials [12-13], and it can be used to observe and measure backward waves in acoustic waveguides.

In this paper, both numerical calculations and the dynamic photoelastic method have been used to study the backward modes in plates.

II. DYNAMIC PHOTOELASTIC SYSTEM

The dynamic photoelastic system is based on the temporary double refraction [13]. The K9 glass is optically isotropic when it is free of stress, but it will turn to optically anisotropic and display the characteristics similar to crystals under stress. The schematic of the system is presented in Figure 1. The light is passed through the polarizer which converts the light into the plane polarized light. And then, this plane polarized light passes through the K9 glass plate and it is splitted into two components along the directions of the first and second principal stresses at each point of the plate. The light is then made to pass through the analyzer, and it brings the information of stress distributions in the plate which will be captured by the camera.

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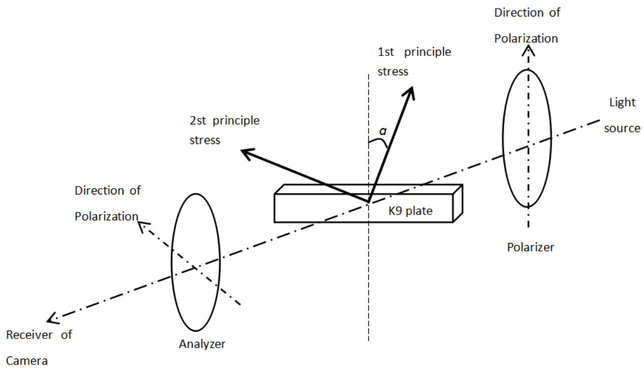


Fig. 1. A schematic diagram of the Photoelastic technique to observe the backward waves in a plate.

III. DISPERSION CURVES OF BACKWARD LAMB MODES

The transparent plate is made of the K9 optical glass. It's longitudinal and shear velocities are V_L = 5957 m/s and V_T = 3608 m/s, respectively. The setting thickness of the plate is $D = 2d$ = 3 mm. The motion is supposed to take place in two dimensions. And the wave is propagating along the *x*-axis. The Rayleigh-Lamb equation in a isotropic plate is given by [14]:

$$\frac{\tanh \beta d}{\tanh \alpha d} = \left(\frac{4\xi^2 \alpha\beta}{\left(\alpha^2 + \beta^2\right)^2} \right)^{\pm 1} \tag{1}$$

in which

$$\alpha^2 = \xi^2 - \lambda_L^2,\ \beta^2 = \xi^2 - \lambda_T^2, \tag{2}$$

Where ξ is the wavenumber along the axis of *x*, $\lambda_L = \omega/V_L$ and $\lambda_T = \omega/V_T$ are wavenumbers of the longitudinal and shear waves of plate in a given frequency, respectively. The dispersion curves of Lamb waves in the K9 glass plate can be calculated from the Rayleigh-Lamb equation (1), and they are displayed in Fig. 2.

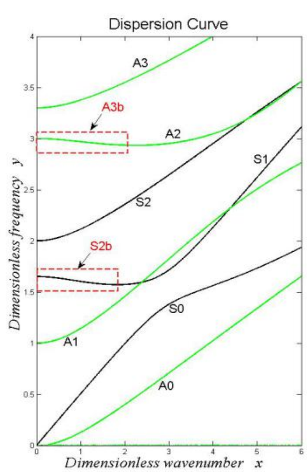


Fig. 2. Lamb wave dispersion curves in the K9 glass plate of thickness 3 mm, with longitudinal and shear wave velocity of 5950 m/s and 3608 m/s, respectively. The green line is the anti-symmetric modes, the black line is the symmetric modes.

In Fig. 2, the dimensionless wavenumber of *x*-axis and dimensionless frequency of *y*-axis are kD and $2fD/V_T$ respectively. Here, V_T is the shear wave velocity of the K9 glass. The phase velocity is $v_p=\omega/k$, and the group velocity is $v_g=d\omega/dk$. The curves encircled by the dashed red block in Fig. 2 are the backward-wave branches S_{2b} and A_{3b}, here the subscript *b* denotes backward wave [15]. For example, the S_{2b} branch extends from the wavenumber k = 0 to the zero-group-velocity (ZGV) point where the slope of the dispersion curve goes to zero. It seems to connect with the mode S1 in the real frequency-wavenumber space, but it is classified as a part of the S2 mode because it is linking to the S2 mode by a pure imaginary loop in the complex frequency-wavenumber space [11]. The slope of the dispersion curve of the mode S_{2b} is negative, that is, the group velocity is opposite to the phase velocity. And the backward mode A_{3b} has the similar dispersion property.

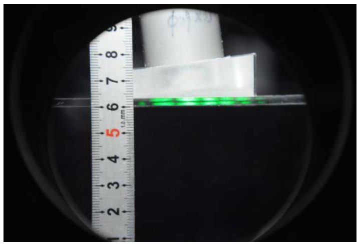
IV. VISUALIZATIONS OF BACKWARD WAVES

Both the symmetric and anti-symmetric backward Lamb modes S_{2b} and A_{3b} in the glass plate have been visualized using the photoelastic system illustrated in Fig. 1. And group velocities of backward modes are measured and compared with the theoretical predictions.

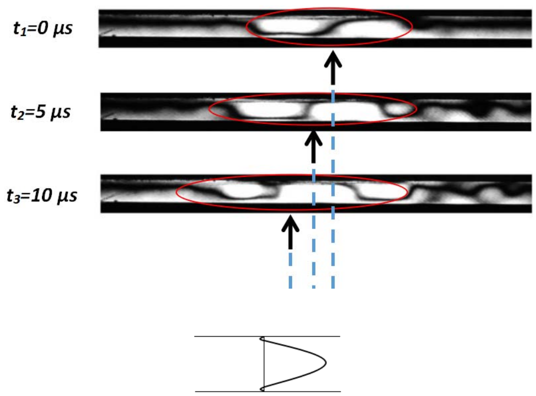
In order to observe the backward mode S_{2b}, the point (x,y) = (0.89, 1.61) in the dispersion curve of S_{2b} in Fig. 2 has been chosen as the excitation point. That is, the product of the frequency and plate thickness is $fD = 2904 Hz{\cdot}m$. The five cycles of continuous waves, excited by the piezoelectric transducer, strikes the glass plate at an angle of 6.5° adjusted by a wedge, consequently, the S_{2b} mode is formed in plate. Moreover, in order to study of propagation characteristics of backward waves and to measure the group velocity, positions of acoustic waves at different time in plate are obtained by adjusting the sound and light time delays. Besides, the photoelastic system is adjusted to display the amplitude distribution of the wave normal stresses $(\sigma_{xx}-\sigma_{yy})/2$ by modulating the direction of the polarization axis of the polarizer and the analyzer.

In the photoelastic images of Fig. 3(b) and 3(c), the stresses of lamb waves at each point in the plate are acquired as absolute values. The consecutive vertical bright lines are the stress peaks or valleys in photoelastic, whereas the dark area has no corresponding stress. From the fig. 3(a), we can infer that the wave front is set to spread to the right, i.e. the direction of the phase velocity is right. While, as shown in Fig. 3(b), the wave package is moving to the left with time increasing, i.e. the direction of the group velocity is left. And the measured group velocity is 708 ± 28m/s. The theoretical group velocity is 739 m/s, which can be calculated from the dispersion curve in Fig.2.

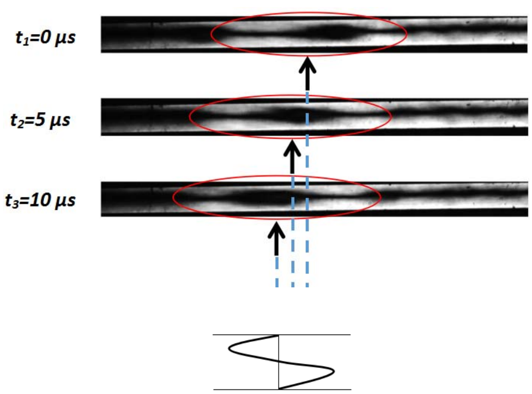
Similarly, in order to excite the A_{3b} mode, the point (x,y) = (0.98, 2.97) in Fig. 2 has been chosen as the excitation point. The incidence angle of the transducer is 3.92°, and the frequency-thickness product is $fD= 5358 Hz{\cdot}m$. The wave front is also set to spread to the right. As showed in Fig. 3(c), the wave flux goes to the left. Besides, the backward-propagation A_{3b} mode has the group velocity of 450 ± 16m/s which is comparable to theoretical prediction, which is 509 m/s.



3(a)



3(b)



3(c)

Fig. 3. Photoelastic images of backward waves. 3(a) is the lamb wave exitation part of dynamic photoelastic system. 3(b) is the normal stress distribution of S_{2b} at $fD = 2904Hz{\cdot}m$, which is symmetric. 3(c) is the normal stress distribution of A_{3b} at $fD = 5358Hz{\cdot}m$, which is anti-symmetric .

V. Conclusions

In this paper, propagation characteristics of backward Lamb modes S_{2b} and A_{3b} in the optical glass (K9) plates have been studied numerically and experimentally. Firstly, the dispersion equation of Lamb modes in the glass plate has been numerically calculated to obtain phase and group velocities dispersion curves. The aims are to identify backward modes with negative group velocities and also to predict phase and group velocities of these modes. Secondly, the dynamic photoelasticity, which is based on stress birefringence, has been used to catch the stress distributions of backward modes S_{2b} and A_{3b} at a given frequency. The propagation of backward modes has been visualized by adjusting the time delays between sound and light. And group velocities are compared with predicted values. However, the images aren't so perfect because of the residual stress in the glass plate. For instance, in Fig. 3(c), the normal stress distribution of A_{3b} is too small to be observed due to the residual stress.

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