Modified Rao Test for Multichannel Adaptive Signal Detection

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***Abstract*—The problem of detecting a subspace signal is studied in colored Gaussian noise with an unknown covariance matrix. In the subspace model, the target signal belongs to a known subspace, but with unknown coordinates. We first present a new derivation of the Rao test based on the subspace model, and then propose a modified Rao test (MRT) by introducing a tunable parameter. The MRT is more general, which includes the Rao test and the generalized likelihood ratio test as special cases. Moreover, closed-form expressions for the probabilities of false alarm and detection of the MRT are derived, which show that the MRT bears a constant false alarm rate property against the noise covariance matrix. Numerical results demonstrate that the MRT can offer the flexibility of being adjustable in the mismatched case where the target signal deviates from the presumed signal subspace. In particular, the MRT provides better mismatch rejection capacities as the tunable parameter increases.**

***Index Terms*—Adaptive detection, constant false alarm rate, mismatched signal rejection, Rao test, subspace signal detection.**

I. Introduction

In recent years, there have been a large number of investigations on the signal detection problem in colored Gaussian noise with an unknown covariance matrix [1]–[9]. Typically, a set of training (secondary) data is assumed to be available to estimate the unknown noise covariance matrix. Many classic detectors have been proposed. For instance, Kelly proposed a generalized likelihood ratio test (GLRT) detector in [10] by replacing the unknown parameters by their maximum likelihood (ML) estimates in the likelihood ratio. To alleviate the computational complexity of the GLRT detector, an adaptive matched filter (AMF) was developed by using a two-step procedure in [11]. Specifically, in the first step of the AMF, it is assumed that the noise covariance matrix is known, and a GLRT is obtained by maximizing the likelihood functions over other unknown parameters. Then, the ML estimate of the noise covariance matrix using the training data alone is substituted into the resulting test obtained from the first step. Another classic detector is the adaptive coherence estimator (ACE) [12], [13] which was obtained by taking into account power non-homogeneity between the test and training data. Note that the GLRT, AMF and ACE are developed for the matched case where the target signal is perfectly matched to the assumed steering vector.

In practice, mismatch of the signal steering vector may exist due to many factors such as wavefront distortions, calibration and pointing errors, and imperfect antenna shape [14]. In [15], Pulsone *et al.* proposed an adaptive beamformer orthogonal rejection test (ABORT) by adding a fictitious signal under the null hypothesis. This fictitious signal is assumed to be orthogonal to the target signal in the whitened observation space. It is shown that the ABORT exhibits better mismatch discrimination capabilities than both the GLRT and AMF. Such capabilities are desired in some practical scenarios. For example, when a target is outside the antenna main beam but is picked up by the sidelobe, it is often desirable to reject the sidelobe target (for the purpose of target localization) and wait until the target enters into the main beam. An ABORT-like detector with improved selectivity for distributed target detection was proposed in [16] and analyzed in [17]. De Maio derived a Rao test in [14], which achieves better rejection capacities of strong mismatched signals than the ABORT. Note that none of the above mentioned detectors can adjust their rejection capabilities of mismatched signals.

In [18], [19], an adaptive sidelobe blanker (ASB) consisting of a cascade of the AMF and ACE was introduced, which can trade a slight loss of detection performance of matched signals for better rejection capabilities of mismatched signals. This cascade approach was also employed in [20]–[23]. Specifically, Pulsone *et al.* developed a computationally efficient two-stage detector consisting of the AMF test followed by the GLRT test [20]. It can achieve a detection and sidelobe rejection performance commensurate with the GLRT, but works in a lower computational complexity. By introducing a multi-rank subspace model for target signals to take into account an uncertainty of the

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signal model, Bandiera *et al.* presented a subspace-based adaptive sidelobe blanker (S-ASB) in [21], and a whitened ABORT subspace-based ASB (WAS-ASB) in [22]. The former consists of the subspace-version GLRT followed by the ACE, whereas the latter is the cascade of the subspace-version GLRT and the whitened ABORT. The selectivity of these two-stage detectors can be adjusted by choosing different threshold pairs. Note that in the S-ASB and WAS-ASB, the subspace model is adopted *only* in the first stage (i.e., the subspace-version GLRT), which can lead to an improvement in robustness. Another approach on addressing target uncertainty is to employ convex constraints for the target steering vector, as used in [24]–[26] to seek detection robustness.

In the above detectors, the target steering vector is a rank-one signal. It is noted that the usage of a multi-rank subspace model in the first stage of the S-ASB or WAS-ASB aims to account for an uncertainty of the actual signal steering vector; however, the target signal is still rank-one. In some applications, the signal of interest is naturally multi-rank. For example, the data collected from multiple polarimetric channels in polarization radars can be formulated as a subspace model for target detection [27]–[34]. It should be stressed that in such cases, the subspace model is not intended to account for target uncertainty. The subspace signal model was also employed for multiuser detection [35]–[37], and signal estimation in multipath environments [38], [39]. The subspace signal detection problem has been extensively studied in, e.g., [40]–[43]. The GLRT and AMF were extended to a rank-two subspace model in [27] and [29], respectively. In [44], the performance of the GLRT and AMF was evaluated analytically for the subspace model where the subspace dimension is arbitrary. An adaptive subspace detector (ASD) was proposed in [45] as a generalization of the ACE. Recently, Liu *et al.* generalized the Rao test from the rank-1 to rank-r ($r > 1$) subspace signal model, and obtained a subspace version of the Rao test in [46]. However, the theoretical performance of the subspace-version Rao test was not examined. Moreover, one common issue in the detectors mentioned above is that the detection performance for matched signals and rejection performance for mismatched signals cannot be adjusted when the target signal has multi-rank. In practice, it is desired to offer a tradeoff between the two performance metrics for matched and, respectively, mismatched signals.

In this paper, we examine the subspace signal detection problem whereby the signal of interest is constrained to a multi-rank subspace with unknown coordinates. Our main contributions are listed as below:

1) We provide a simple derivation of the subspace Rao test by using both the test and training data in the estimation of the noise covariance matrix. This derivation offers an additional insight that the detection probability of the subspace Rao test does not necessarily increase with the signal-to-noise ratio (SNR).
2) A new modified Rao test (MRT) with a tunable parameter is proposed, which includes the GLRT and Rao test as special cases. Our proposed MRT is notably different from the two tunable detectors introduced in [23], [47], which were designed for rank-one signal detection but cannot be used for multi-rank subspace signal detection. Numerical results show that the mismatched signal rejection performance of the proposed MRT improves as the tunable parameter increases. Remarkably, the MRT with a large tunable parameter can better reject mismatched signals than existing detectors.
3) We develop theoretical results pertaining to the statistical properties of the MRT for both the matched and mismatched cases. In the matched case, the target signal belongs to a presumed subspace, whereas in the mismatched case the target signal deviates from the presumed subspace. Closed-form expressions for the probabilities of false alarm and, respectively, detection of the MRT are derived under a non-fluctuating as well as a fluctuating target model. Our theoretical results are confirmed by Monte Carlo (MC) simulations. It is found that the MRT has a constant false alarm rate (CFAR) with respect to the noise covariance matrix. These theoretical expressions facilitate the performance evaluation of the MRT in practical scenarios.

The remainder of this paper is organized as follows. Section II contains the signal model. In Section III, the Rao test is introduced, and a simple derivation of the Rao test is provided. A modified Rao test is proposed, and its performance analysis is included in Section IV. Simulation results are presented in Section V, and finally the paper is concluded in Section VI.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters. Superscripts $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^\dagger$ denote transpose, complex conjugate and complex conjugate transpose, respectively. The notation $\sim$ means "is distributed as," and $\mathcal{CN}$ denotes a circularly symmetric, complex Gaussian distribution. $\mathrm{E}\{\cdot\}$ denotes the mean of a random argument. $\overset{\mathrm{d}}{=}$ means equivalence in distribution. χ_n^2 denotes the central Chi-squared distribution with n degrees of freedom, while $\chi_n'^2(\zeta)$ denotes the non-central Chi-squared distribution with n degrees of freedom and a non-centrality parameter ζ. $|\cdot|$ represents the modulus of a complex number, and $\jmath = \sqrt{-1}$. $\binom{n}{m}$ is the binomial coefficient. $\mathbf{I}_n$ is the identity matrix of dimension n, and $\mathrm{tr}(\cdot)$ is the trace of a matrix.

II. Signal Model

Consider the following model of the test data:

$$\mathbf{x} = \mathbf{S}\mathbf{a} + \mathbf{n}, \tag{1}$$

where $\mathbf{S}$ is a known full-rank matrix of dimension $Q \times q$ whose columns span the subspace containing target signals; $\mathbf{a}$ is a deterministic but unknown coordinate vector of dimension q, accounting for the target reflectivity and channel propagation effects; the noise $\mathbf{n}$ is assumed to have a circularly symmetric, complex Gaussian distribution, i.e., $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$, where $\mathbf{R}$ is a positive definite covariance matrix of dimension $Q \times Q$. In practice, the noise covariance matrix $\mathbf{R}$ is usually unknown. A standard assumption is that there exists a set of homogeneous training data free of target signal components, i.e., $\{\mathbf{y}_k | \mathbf{y}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}),\ k = 1, 2, \ldots, K \text{ and } K \geq Q\}$, which can be used to estimate $\mathbf{R}$.

Example: The model (1) is often employed in polarimetric radars where the data received in multiple polarimetric channels are combined for target detection [27]–[29]. More specifically, assume that a polarimetric radar transmits a train of M radar pulses alternatively at each of two linear polarizations (i.e., H and V) and receives in both. The available polarimetric channels are HH, VV, VH, and HV. In the ith polarimetric channel, the received data is denoted by

$$\mathbf{x}_i = \mathbf{s}a_i + \mathbf{n}_i \tag{2}$$

where a_i is the complex target amplitude, and $\mathbf{s}$ is the signal steering vector represented by

$$\mathbf{s} = \frac{1}{\sqrt{M}}\left[1, \exp(\jmath 2\pi f_{\mathrm{d}}), \cdots, \exp\left(\jmath 2\pi (M-1) f_{\mathrm{d}}\right)\right]^T \tag{3}$$

with f_{d} being the normalized target Doppler frequency. Suppose that we use the data collected from q polarimetric channels for target detection (e.g., HH and VV channels are employed if $q = 2$). Stacking these data $\mathbf{x}_1, \ldots, \mathbf{x}_q$ into a longer column vector $\mathbf{x}$ of dimension Mq, we have

$$\begin{aligned}\mathbf{x} &= \left[\mathbf{x}_1^T, \ldots, \mathbf{x}_q^T\right]^T \\ &= \mathbf{S}\mathbf{a} + \mathbf{n},\end{aligned} \tag{4}$$

where $\mathbf{S} = \mathbf{I}_q \otimes \mathbf{s} \in \mathbb{C}^{Q\times q}$ with $Q = Mq$, $\mathbf{a} = [a_1, \ldots, a_q]^T \in \mathbb{C}^q$, and $\mathbf{n} = \left[\mathbf{n}_1^T, \ldots, \mathbf{n}_q^T\right]^T \in \mathbb{C}^Q$. The covariance matrix of $\mathbf{n}$ has the block structure

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_{\frac{1}{1}} & \cdots & \mathbf{R}_{\frac{1}{q}} \\ \vdots & \ddots & \vdots \\ \mathbf{R}_{\frac{q}{1}} & \cdots & \mathbf{R}_{\frac{q}{q}} \end{bmatrix}, \tag{5}$$

where each $M \times M$ block matrix $\mathbf{R}_{n/m}, n, m = 1, 2, \cdots, q$ denotes the correlation property of a pair of polarimetric channels. When $q = 2$,

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_{\frac{\mathrm{HH}}{\mathrm{HH}}} & \mathbf{R}_{\frac{\mathrm{HH}}{\mathrm{VV}}} \\ \mathbf{R}_{\frac{\mathrm{VV}}{\mathrm{HH}}} & \mathbf{R}_{\frac{\mathrm{VV}}{\mathrm{VV}}} \end{bmatrix} = \sigma^2 \begin{bmatrix} \mathbf{C} & \nu\mathbf{C} \\ \nu\mathbf{C} & \mathbf{C} \end{bmatrix} \tag{6}$$

where σ^2 denotes the noise power in the HH or VV polarimetric channel, ν is a fraction accounting for the cross-correlation between the two copolarized noise, and $\mathbf{C}$ is the normalized noise covariance matrix [28].

Let the null hypothesis (H_0) be that the test data are target signal free and the alternative hypothesis (H_1) be that the test data contain the target signal. Hence, the detection problem is to decide between the null hypothesis

$$H_0 : \begin{cases} \mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}) \\ \mathbf{y}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}),\ k = 1, \cdots, K, \end{cases} \tag{7a}$$

and the alternative one

$$H_1 : \begin{cases} \mathbf{x} \sim \mathcal{CN}(\mathbf{S}\mathbf{a}, \mathbf{R}) \\ \mathbf{y}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}),\ k = 1, \cdots, K. \end{cases} \tag{7b}$$

It has to be emphasized here that for $q > 1$ the target signal in the received data $\mathbf{x}$ intrinsically belongs to a subspace spanned by the columns of $\mathbf{S}$, and this multi-rank subspace model is not for the purpose of accounting for the uncertainty on the signal steering vector as in [21], [22]. As such, the two-stage detectors such as the S-ASB and WAS-ASB developed therein cannot be applied to the multi-rank subspace detection problem with $q > 1$ in (7), since they are all designed for rank-one signal detection. It is worth noting that the second test in the S-ASB (or WAS-ASB) is the ACE (or whitened ABORT) which is for rank-one signal detection. In addition, the tunable detectors proposed in [23], [47] cannot either be used in the detection problem (7) with $q > 1$.

III. Rao Test

For the above detection problem (7), several adaptive detectors have been proposed, such as the subspace-version GLRT in [27], subspace-version AMF in [29], and subspace-version Rao test in [46]. Here, we reconsider the Rao test. We first briefly review the standard approach leading to the Rao test, and then give a new and simpler derivation for the Rao test, which offers some useful insight into the detector. In the next section, we will propose a new tunable detector which includes the GLRT and Rao test as special cases.

A. Prior Work

Let $\boldsymbol{\Theta}$ be the parameter vector partitioned as

$$\boldsymbol{\Theta} = \left[\boldsymbol{\Theta}_{\mathrm{r}}^T, \quad \boldsymbol{\Theta}_{\mathrm{s}}^T\right], \tag{8}$$

where $\boldsymbol{\Theta}_{\mathrm{r}} = \mathbf{a}$ and $\boldsymbol{\Theta}_{\mathrm{s}} = \mathrm{vec}(\mathbf{R})$. Usually, $\boldsymbol{\Theta}_{\mathrm{r}}$ and $\boldsymbol{\Theta}_{\mathrm{s}}$ are called the relative and nuisance parameters, respectively. Denote by f_1 and f_0 the joint probability density functions (PDFs) of $\mathbf{x}$ and $\mathbf{Y} \triangleq [\mathbf{y}_1, \cdots, \mathbf{y}_K]$ under H_1 and H_0, respectively. The Fisher information matrix $\mathbf{F}(\boldsymbol{\Theta})$ associated with f_1 can be expressed as

$$\begin{aligned}\mathbf{F}(\boldsymbol{\Theta}) &= \mathrm{E}\left\{\frac{\partial \ln f_1}{\partial \boldsymbol{\Theta}^*}\frac{\partial \ln f_1}{\partial \boldsymbol{\Theta}^T}\right\} \\ &\triangleq \begin{bmatrix} \mathbf{F}_{\boldsymbol{\Theta}_{\mathrm{r}},\boldsymbol{\Theta}_{\mathrm{r}}} & \mathbf{F}_{\boldsymbol{\Theta}_{\mathrm{r}},\boldsymbol{\Theta}_{\mathrm{s}}} \\ \mathbf{F}_{\boldsymbol{\Theta}_{\mathrm{s}},\boldsymbol{\Theta}_{\mathrm{r}}} & \mathbf{F}_{\boldsymbol{\Theta}_{\mathrm{s}},\boldsymbol{\Theta}_{\mathrm{s}}} \end{bmatrix}.\end{aligned} \tag{9}$$

According to [48], the Rao test for complex-valued signals is given as

$$\Xi = \left.\frac{\partial \ln f_1}{\partial \boldsymbol{\Theta}_{\mathrm{r}}}\right|^T_{\boldsymbol{\Theta}=\hat{\boldsymbol{\Theta}}_0} \left[\mathbf{F}^{-1}(\hat{\boldsymbol{\Theta}}_0)\right]_{\boldsymbol{\Theta}_{\mathrm{r}},\boldsymbol{\Theta}_{\mathrm{r}}} \left.\frac{\partial \ln f_1}{\partial \boldsymbol{\Theta}_{\mathrm{r}}^*}\right|_{\boldsymbol{\Theta}=\hat{\boldsymbol{\Theta}}_0}, \tag{10}$$

where $\hat{\boldsymbol{\Theta}}_0$ is the ML estimate of $\boldsymbol{\Theta}$ under H_0, and $[\mathbf{F}^{-1}(\hat{\boldsymbol{\Theta}}_0)]_{\boldsymbol{\Theta}_{\mathrm{r}},\boldsymbol{\Theta}_{\mathrm{r}}}$ is

$$\left[\mathbf{F}^{-1}(\hat{\boldsymbol{\Theta}})\right]_{\boldsymbol{\Theta}_{\mathrm{r}},\boldsymbol{\Theta}_{\mathrm{r}}} = \left[\mathbf{F}_{\boldsymbol{\Theta}_{\mathrm{r}},\boldsymbol{\Theta}_{\mathrm{r}}} - \mathbf{F}_{\boldsymbol{\Theta}_{\mathrm{r}},\boldsymbol{\Theta}_{\mathrm{s}}}\mathbf{F}^{-1}_{\boldsymbol{\Theta}_{\mathrm{s}},\boldsymbol{\Theta}_{\mathrm{s}}}\mathbf{F}_{\boldsymbol{\Theta}_{\mathrm{s}},\boldsymbol{\Theta}_{\mathrm{r}}}\right]^{-1} \tag{11}$$

evaluated at $\hat{\boldsymbol{\Theta}}_0$.

Define

$$\hat{\mathbf{R}} = \sum_{k=1}^{K} \mathbf{y}_k \mathbf{y}_k^\dagger. \tag{12}$$

As derived in [46], the Rao test for the detection problem in (7) can be expressed as

$$\Psi = \frac{\tilde{\mathbf{x}}^\dagger \mathbf{P}_{\tilde{\mathbf{S}}} \tilde{\mathbf{x}}}{(1 + \tilde{\mathbf{x}}^\dagger\tilde{\mathbf{x}})\left(1 + \tilde{\mathbf{x}}^\dagger \mathbf{P}^{\perp}_{\tilde{\mathbf{S}}}\tilde{\mathbf{x}}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \psi, \tag{13}$$

where

$$\tilde{\mathbf{S}} = \hat{\mathbf{R}}^{-1/2}\mathbf{S}, \tag{14}$$

$$\tilde{\mathbf{x}} = \hat{\mathbf{R}}^{-1/2}\mathbf{x} \tag{15}$$

with $(\cdot)^{-1/2}$ denoting the inverse of the Hermitian square root of a matrix, and

$$\mathbf{P}_{\tilde{\mathbf{S}}} = \tilde{\mathbf{S}}(\tilde{\mathbf{S}}^\dagger\tilde{\mathbf{S}})^{-1}\tilde{\mathbf{S}}^\dagger, \tag{16}$$

$$\mathbf{P}_{\tilde{\mathbf{S}}}^\perp = \mathbf{I} - \tilde{\mathbf{S}}(\tilde{\mathbf{S}}^\dagger\tilde{\mathbf{S}})^{-1}\tilde{\mathbf{S}}^\dagger. \tag{17}$$

B. A Simple Derivation of the Rao Test

Here, we give a new derivation of the Rao test (13) by modifying the classic AMF detector which is given by

$$T_{\rm AMF} = \mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S}(\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S})^{-1}\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x} \underset{H_0}{\overset{H_1}{\gtrless}} t_{\rm AMF}, \tag{18}$$

where $t_{\rm AMF}$ is the detection threshold, and $\hat{\mathbf{R}}$ is defined in (12).

We first form an estimate of the noise covariance matrix by exploiting both the test and training data,

$$\bar{\mathbf{R}} = \mathbf{x}\mathbf{x}^\dagger + \sum_{k=1}^{K}\mathbf{y}_k\mathbf{y}_k^\dagger = \mathbf{x}\mathbf{x}^\dagger + \hat{\mathbf{R}}. \tag{19}$$

Then, a modified AMF detector is obtained by using $\bar{\mathbf{R}}$ to replace $\hat{\mathbf{R}}$ in the classic AMF, i.e.,

$$\Xi_{\rm Rao} = \mathbf{x}^\dagger\bar{\mathbf{R}}^{-1}\mathbf{S}(\mathbf{S}^\dagger\bar{\mathbf{R}}^{-1}\mathbf{S})^{-1}\mathbf{S}^\dagger\bar{\mathbf{R}}^{-1}\mathbf{x} \underset{H_0}{\overset{H_1}{\gtrless}} \xi_{\rm Rao}, \tag{20}$$

where $\xi_{\rm Rao}$ is a detection threshold. In the sequel, we prove the equivalence between (13) and (20).

According to the matrix inversion lemma [49, p. 1348], we have

$$\bar{\mathbf{R}}^{-1} = \hat{\mathbf{R}}^{-1} - \frac{\hat{\mathbf{R}}^{-1}\mathbf{x}\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}}{1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}}. \tag{21}$$

Therefore,

$$\mathbf{x}^\dagger\bar{\mathbf{R}}^{-1}\mathbf{S} = \frac{\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S}}{1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}}, \tag{22}$$

and

$$\mathbf{S}^\dagger\bar{\mathbf{R}}^{-1}\mathbf{S} = \mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S} - \frac{\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S}}{1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}}. \tag{23}$$

Taking (22) and (23) into the test statistic $\Xi_{\rm Rao}$ of (20), we obtain

$$\begin{aligned}\Xi_{\rm Rao} &= \frac{\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S}}{1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}}\left[\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S} - \frac{\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S}}{1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}}\right]^{-1} \\ &\quad\times\frac{\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}}{1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}} \\ &= \frac{\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S}\mathbf{W}^{-1}\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}}{1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}},\end{aligned} \tag{24}$$

where

$$\mathbf{W} = (1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x})(\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S}) - \mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S}. \tag{25}$$

Applying the matrix inversion lemma to (25), we have

$$\mathbf{W}^{-1} = \frac{\frac{(\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S})^{-1}}{1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}}\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S}\frac{(\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S})^{-1}}{1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}}}{1-\frac{\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S}(\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S})^{-1}\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}}{1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}}} + \frac{(\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S})^{-1}}{1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}}. \tag{26}$$

Substituting (26) into (24) leads to

$$\begin{aligned}\Xi_{\rm Rao} &= \frac{\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S}(\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S})^{-1}\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}}{(1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x})^2} \\ &\quad\times\left[1+\frac{\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S}(\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S})^{-1}\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}}{1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}-\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S}(\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S})^{-1}\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}}\right] \\ &= \frac{T_{\rm AMF}}{(1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x})(1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}-T_{\rm AMF})},\end{aligned} \tag{27}$$

where $T_{\rm AMF}$ is given in (18). Using the notations in (14)-(17), we have

$$T_{\rm AMF} = \tilde{\mathbf{x}}^\dagger\mathbf{P}_{\tilde{\mathbf{S}}}\tilde{\mathbf{x}}, \quad \mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x} = \tilde{\mathbf{x}}^\dagger\tilde{\mathbf{x}} \tag{28}$$

and

$$\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x} - T_{\rm AMF} = \tilde{\mathbf{x}}^\dagger\mathbf{P}_{\tilde{\mathbf{S}}}^\perp\tilde{\mathbf{x}}. \tag{29}$$

Thus, $\Xi_{\rm Rao}$ in (27) can be rewritten as the Rao test Ψ defined in (13). This shows that the detector in (20) is equivalent to the Rao test in (13), even though they have different forms.

The above new derivation reveals that in the Rao test, the estimate of the noise covariance matrix is obtained by using the training data and as well the test data. Note that the test data may include the target signal, in which case it leads to the signal contamination in the noise covariance matrix estimate. When the number of training data is limited, the effect of the signal contamination on the noise covariance matrix estimate is more severe. It is expected that the detection performance of the Rao test degrades in the case of limited training data. This prediction will be confirmed by simulation results in Section V.

It is worth noting that in practice, the Rao test in the form of (20) has considerably less computational complexity than that in (13) when running the detector continuously over a block of range bins. Specifically, for (20), the covariance matrix estimate (19) is computed only once by using all data inside the data block. To the contrary, (13) requires a different covariance matrix to be computed for each range bin, by removing the test data and using the rest of the data block along with (12) for covariance matrix estimation.

IV. Modified Rao Test

As shown above, the Rao test has the following form:

$$\Xi_{\rm Rao} = \frac{T_{\rm AMF}}{(1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x})(1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}-T_{\rm AMF})} \underset{H_0}{\overset{H_1}{\gtrless}} \xi_{\rm Rao}. \tag{30}$$

Recall that the GLRT detector proposed in [27], [44] can be written as

$$T_{\rm GLRT} = \frac{T_{\rm AMF}}{1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}} \underset{H_0}{\overset{H_1}{\gtrless}} t_{\rm GLRT}, \tag{31}$$

where t_{GLRT} is the detection threshold. Note that the only difference between the Rao test and the GLRT is the second term $(1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}-T_{\mathrm{AMF}})$ in the denominator of (30). Based on this observation, we propose a modified Rao test (MRT) involving a tunable parameter as follows:

$$\begin{aligned}\Xi &= \frac{T_{\mathrm{AMF}}}{(1+\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x})\left[1+\alpha(\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}-T_{\mathrm{AMF}})\right]}\\ &= \frac{\tilde{\mathbf{x}}^\dagger\mathbf{P}_{\tilde{\mathbf{S}}}\tilde{\mathbf{x}}}{(1+\tilde{\mathbf{x}}^\dagger\tilde{\mathbf{x}})\left(1+\alpha\tilde{\mathbf{x}}^\dagger\mathbf{P}_{\tilde{\mathbf{S}}}^{\perp}\tilde{\mathbf{x}}\right)}\\ &\underset{H_0}{\overset{H_1}{\gtrless}} \xi, \end{aligned} \tag{32}$$

where ξ is a detection threshold, $\alpha \geq 0$ is a tunable parameter. Obviously, the MRT contains the GLRT and Rao test as special cases of $\alpha = 0$ and $\alpha = 1$, respectively.

It should be pointed out that the analytical performance of the Rao test is not examined in [46]. In the sequel, we first investigate the statistical properties of the proposed MRT, and then derive closed-form expressions for its probabilities of false alarm and detection. Apparently, by setting $\alpha = 1$ we also fill the gap on the analytical performance of the Rao test that is missing in [46].

Similar to [40], it can be shown that

$$\Xi = \frac{T_{\mathrm{AMF}}}{(\rho^{-1}+T_{\mathrm{AMF}})\left[1+\alpha(\rho^{-1}-1)\right]} \underset{H_0}{\overset{H_1}{\gtrless}} \xi, \tag{33}$$

where ρ is a loss factor whose PDF is

$$f_\rho(\rho) = \frac{K!(1-\rho)^{Q-q-1}\rho^{K-Q+q}}{(Q-q-1)!(K-Q+q)!}, \quad 0<\rho<1. \tag{34}$$

After an equivalent transformation, we have

$$\rho T_{\mathrm{AMF}} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\xi(\rho+\alpha-\rho\alpha)}{\rho-\xi(\rho+\alpha-\rho\alpha)}. \tag{35}$$

Similar to [50, eq. (B39)], we can derive that

$$\rho T_{\mathrm{AMF}} \overset{\mathrm{d}}{=} \begin{cases} \dfrac{\chi^2_{2q}}{\chi^2_{2(K-Q+1)}}, & \text{under } H_0\\ \dfrac{\chi'^2_{2q}(2\delta\rho)}{\chi^2_{2(K-Q+1)}}, & \text{under } H_1 \end{cases} \tag{36}$$

with

$$\delta = \mathbf{a}^\dagger\mathbf{S}^\dagger\mathbf{R}^{-1}\mathbf{S}\mathbf{a}. \tag{37}$$

Note that to guarantee the positiveness of the right-hand side of (35), the value of the random variable ρ is now restricted to the range

$$\frac{\xi\alpha}{1-\xi+\xi\alpha} < \rho < 1. \tag{38}$$

Define

$$2\tau \overset{\mathrm{d}}{=} \chi^2_{2(K-Q+1)}, \tag{39}$$

and

$$2t \overset{\mathrm{d}}{=} \begin{cases} \chi^2_{2q}, & \text{under } H_0\\ \chi'^2_{2q}(2\delta\rho), & \text{under } H_1 \end{cases}. \tag{40}$$

Then, we obtain

$$\rho T_{\mathrm{AMF}} \overset{\mathrm{d}}{=} \frac{t}{\tau}. \tag{41}$$

A. Probability of False Alarm

Based on (35), we can obtain the probability of false alarm conditioned on ρ as

$$\begin{aligned} P_{\mathrm{FA}|\rho} &= \int_0^{+\infty}\left(\int_{\frac{\xi(\rho+\alpha-\rho\alpha)}{\rho-\xi(\rho+\alpha-\rho\alpha)}\tau}^{+\infty} f_t(t|H_0)\mathrm{d}t\right) f_\tau(\tau|H_0)\mathrm{d}\tau\\ &= \sum_{j=1}^{q}\binom{K-Q+q-j}{q-j}\left[\frac{\xi(\rho+\alpha-\rho\alpha)}{\rho-\xi(\rho+\alpha-\rho\alpha)}\right]^{q-j}\\ &\quad\times\left[\frac{\rho}{\rho-\xi(\rho+\alpha-\rho\alpha)}\right]^{-(K-Q+q-j+1)}. \end{aligned} \tag{42}$$

Therefore, the probability of false alarm of the Rao test can be obtained by averaging over ρ, i.e.,

$$P_{\mathrm{FA}} = \int_{\frac{\xi\alpha}{1-\xi+\xi\alpha}}^{1} P_{\mathrm{FA}|\rho} f_\rho(\rho)\mathrm{d}\rho, \tag{43}$$

where $f_\rho(\rho)$ is given in (34). This integral expression for the probability of false alarm can be written as a finite-form form, i.e.,

$$\begin{aligned} P_{\mathrm{FA}} &= \frac{K!}{(Q-q-1)!(K-Q+q)!}\sum_{j=1}^{q}\binom{K-Q+q-j}{q-j}\xi^{q-j}\\ &\times\sum_{n=0}^{K-Q+1}\binom{K-Q+1}{n}(-\xi\alpha)^{K-Q-n+1}(1-\xi+\xi\alpha)^n\\ &\times\sum_{m=0}^{Q-q-1}\binom{Q-q-1}{m}(-1)^m\sum_{k=0}^{q-j}\binom{q-j}{k}\alpha^{q-j-k}\\ &\times\frac{(1-\alpha)^k}{m+n+j+k}\left[1-\left(\frac{\xi\alpha}{1-\xi+\xi\alpha}\right)^{m+n+j+k}\right]. \end{aligned} \tag{44}$$

It follows that the MRT exhibits the desirable CFAR property against the noise covariance matrix, since the probability of false alarm in (44) is irrelevant to the noise covariance matrix.

B. Detection Probability

In this subsection, we will derive the detection probability of the MRT both in matched and mismatched cases. In the matched case the target signal exactly lies in the nominal subspace, whereas in the mismatched case the target signal does not belong to the nominal subspace.

1) Matched Case: We first consider the matched case. Let

$$\omega = \frac{\xi(\rho+\alpha-\rho\alpha)}{\rho-\xi(\rho+\alpha-\rho\alpha)}\tau, \tag{45}$$

where the loss factor ρ is temporarily fixed. The PDF of ω conditioned on ρ is

$$f_{\omega|\rho}(\omega) = \frac{1}{(K-Q)!}\frac{\rho-\xi(\rho+\alpha-\rho\alpha)}{\xi(\rho+\alpha-\rho\alpha)} \times \left[\frac{\rho-\xi(\rho+\alpha-\rho\alpha)}{\xi(\rho+\alpha-\rho\alpha)}\omega\right]^{K-Q} \times \exp\left[-\frac{\rho-\xi(\rho+\alpha-\rho\alpha)}{\xi(\rho+\alpha-\rho\alpha)}\omega\right], \quad (46)$$

where $\omega > 0$. Using (41) and (45), we can rewrite (35) as

$$t \underset{H_0}{\overset{H_1}{\gtrless}} \omega. \quad (47)$$

As a result, the probability of detection conditioned on ρ can be obtained as

$$\begin{aligned} P_{\mathrm{D}|\rho} &= \int_0^{+\infty}\left(\int_\omega^{+\infty} f_t(t|H_1)\mathrm{d}t\right) f_{\omega|\rho}(\omega)\mathrm{d}\omega \\ &= 1-\left[\frac{\xi(\rho+\alpha-\rho\alpha)}{\rho-\xi(\rho+\alpha-\rho\alpha)}\right]^{q-1} \\ &\quad\times\left[\frac{\rho}{\rho-\xi(\rho+\alpha-\rho\alpha)}\right]^{-(K-Q+q)} \\ &\quad\times\sum_{j=1}^{K-Q+1}\binom{K-Q+q}{q+j-1}\left[\frac{\xi(\rho+\alpha-\rho\alpha)}{\rho-\xi(\rho+\alpha-\rho\alpha)}\right]^j \\ &\quad\times\exp\{-\delta[\rho-\xi(\rho+\alpha-\rho\alpha)]\} \\ &\quad\times\sum_{m=0}^{j-1}\frac{\delta^m}{m!}[\rho-\xi(\rho+\alpha-\rho\alpha)]^m. \end{aligned} \quad (48)$$

Furthermore, the detection probability of the MRT is obtained by averaging over ρ, i.e.,

$$P_{\mathrm{D}} = \int_{\frac{\xi\alpha}{1-\xi+\xi\alpha}}^{1} P_{\mathrm{D}|\rho} f_\rho(\rho)\mathrm{d}\rho, \quad (49)$$

where $f_\rho(\rho)$ is given in (34).

For the Rao test (i.e., $\alpha = 1$), we can simplify the integral in (49) as a finite-sum form as follows

$$P_{\mathrm{D}} = V_1 - V_2, \quad (50)$$

where

$$\begin{aligned} V_1 &= \frac{K!}{(Q-q-1)!(K-Q+q)!} \times\int_\xi^1(1-\rho)^{Q-q-1}\rho^{K-Q+q}\mathrm{d}\rho \\ &= \frac{K!}{(Q-q-1)!(K-Q+q)!}\sum_{l=0}^{Q-q-1}\binom{Q-q-1}{l} \\ &\quad\times\frac{(-1)^l}{K-Q+q+l+1} \times\left[1-\xi^{K-Q+q+l+1}\right]. \end{aligned} \quad (51)$$

and

$$\begin{aligned} V_2 &= \frac{K!\exp(\delta\xi)}{(Q-q-1)!(K-Q+q)!}\sum_{j=1}^{K-Q+1}\binom{K-Q+q}{q+j-1} \\ &\quad\times\xi^{q+j-1}\sum_{m=0}^{j-1}\frac{1}{m!}\delta^m \\ &\quad\times\sum_{n=0}^{K-Q-j+m+1}\binom{K-Q-j+m+1}{n} \\ &\quad\times(-\xi)^{K-Q-j+m-n+1}\sum_{k=0}^{Q-q-1}\binom{Q-q-1}{k} \\ &\quad\times(-1)^k\delta^{-(n+k+1)} \\ &\quad\times\left[\gamma(n+k+1,\delta)-\gamma(n+k+1,\delta\xi)\right], \end{aligned} \quad (52)$$

with $\gamma(N, x)$ denoting the incomplete Gamma function defined by [51, p. 899]

$$\begin{aligned} \gamma(N,x) &= \int_0^x y^{N-1}\exp(-y)\mathrm{d}y \\ &= (N-1)!\left[1-\exp(-x)\sum_{i=0}^{N-1}\frac{x^i}{i!}\right], \end{aligned} \quad (53)$$

for arbitrary positive integer N. Up to now, we have derived the finite-sum expressions for the probabilities of false alarm and detection of the Rao test proposed in [46].

Note that the above analysis is based on the un-fluctuating target model where the target signature $\mathbf{a}$ is assumed to be deterministic. In the following, we derive the detection probability of the MRT in the fluctuating target model where the target signature $\mathbf{a}$ is stochastic and has a complex circular Gaussian distribution with zero mean and covariance matrix $\mathbf{R}_{\mathbf{a}}$, i.e., $\mathbf{a} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_{\mathbf{a}})$.

The probability of false alarm of the MRT in the fluctuating target model is the same as that in the un-fluctuating target model, since it is assumed that the target echoes do not exist under H_0. The randomness of the target signature affects the detection probability of the MRT through the non-centrality parameter δ in (37). Hence, we need to derive the PDF of δ, which can be obtained in a way similar to that in [52, eq. (27)].

As an example, we give an explicit expression for the detection probability of the MRT with $q = 2$ in the fluctuating target model. In the case of $q = 2$, we can decompose $\mathbf{S}^\dagger\mathbf{R}^{-1}\mathbf{S}$ as

$$\mathbf{S}^\dagger\mathbf{R}^{-1}\mathbf{S} = \eta_1\mathbf{u}_1\mathbf{u}_1^\dagger + \eta_2\mathbf{u}_2\mathbf{u}_2^\dagger, \quad (54)$$

where η_j and $\mathbf{u}_j$ are the jth eigenvalue and the corresponding unit-norm eigenvector of $\mathbf{S}^\dagger\mathbf{R}^{-1}\mathbf{S}$, respectively, and it is assumed that $\eta_1 \neq \eta_2$. Let

$$\zeta_j = \mathbf{u}_j^\dagger\mathbf{R}_{\mathbf{a}}\mathbf{u}_j, \quad \text{and} \quad \psi_j = \frac{\zeta_j\eta_j}{2}, \quad j = 1, 2. \quad (55)$$

Then,

$$\delta = \mathbf{a}^\dagger\mathbf{S}^\dagger\mathbf{R}^{-1}\mathbf{S}\mathbf{a} = \psi_1|\mu_1|^2 + \psi_1|\mu_2|^2, \quad (56)$$

where

$$\mu_j = \sqrt{\frac{2}{\zeta_j}}\mathbf{u}_j^\dagger \mathbf{a} \sim \mathcal{CN}(0,2). \tag{57}$$

It follows that the PDF of δ is

$$f_\delta(\delta) = \frac{1}{2(\psi_1 - \psi_2)}\left[\exp\left(-\frac{\delta}{2\psi_1}\right) - \exp\left(-\frac{\delta}{2\psi_2}\right)\right]. \tag{58}$$

Further, the detection probability of the MRT with $q=2$ in the fluctuating target model is

$$P_\mathrm{D} = \int_{\frac{\xi\alpha}{1-\xi+\xi\alpha}}^{1} \underbrace{\int_0^\infty P_{\mathrm{D}|\rho} f_\delta(\delta)\mathrm{d}\delta\, f_\rho(\rho)\mathrm{d}\rho}_{W(\rho)}, \tag{59}$$

where

$$\begin{aligned} W(\rho) =& 1 - \frac{\xi(\rho+\alpha-\rho\alpha)}{\rho - \xi(\rho+\alpha-\rho\alpha)}\left[\frac{\rho}{\rho-\xi(\rho+\alpha-\rho\alpha)}\right]^{-(K-Q+2)} \\ &\times \sum_{j=1}^{K-Q+1}\binom{K-Q+2}{j+1}\left[\frac{\xi(\rho+\alpha-\rho\alpha)}{\rho-\xi(\rho+\alpha-\rho\alpha)}\right]^j \\ &\times \sum_{m=0}^{j-1}\frac{[\rho-\xi(\rho+\alpha-\rho\alpha)]^m}{2(\psi_1-\psi_2)} \\ &\times \left\{\left[\rho-\xi(\rho+\alpha-\rho\alpha)+\frac{1}{2\psi_1}\right]^{-(m+1)}\right. \\ &\left. - \left[\rho-\xi(\rho+\alpha-\rho\alpha)+\frac{1}{2\psi_2}\right]^{-(m+1)}\right\}. \end{aligned} \tag{60}$$

2) Mismatched Case: Here we consider the mismatched case where the actual target signal subspace deviates from the presumed subspace. To quantify the mismatching, we define the angle ϕ between the actual signal steering vector $\mathbf{S}_0$ and the nominal subspace $\mathbf{S}$ as follows [46]

$$\cos^2\phi = \frac{\left|\mathrm{tr}(\mathbf{S}^\dagger\mathbf{R}^{-1}\mathbf{S}_0)\right|^2}{\mathrm{tr}\left(\mathbf{S}_0^\dagger\mathbf{R}^{-1}\mathbf{S}_0\right)\mathrm{tr}(\mathbf{S}^\dagger\mathbf{R}^{-1}\mathbf{S})}. \tag{61}$$

Note that $\phi = 0$ corresponds to the case where the actual signal belongs to the nominal subspace. For the case of $q=1$. the subspace matrix $\mathbf{S}$ reduces to a steering vector denoted by $\mathbf{s}$, and the coordinate vector $\mathbf{a}$ becomes a scalar denoted by a. The angle ϕ between the actual signal steering vector $\mathbf{s}_0$ and the nominal subspace $\mathbf{s}$ becomes

$$\cos^2\phi = \frac{\left|\mathbf{s}_0^\dagger\mathbf{R}^{-1}\mathbf{s}\right|^2}{(\mathbf{s}^\dagger\mathbf{R}^{-1}\mathbf{s})(\mathbf{s}_0\mathbf{R}^{-1}\mathbf{s}_0)}. \tag{62}$$

In the following, we derive a closed-form expression for the detection probability of the MRT for the mismatched case with $q=1$. It is obtained in [53] that the PDF of ρ in the mismatched case is

$$f_\rho^{\mathrm{mis}}(\rho) = \exp(-\rho\Psi_\phi)\sum_{n=0}^{K-Q+2}\binom{K-Q+2}{n}\frac{K!}{(K+n)!} \times\Psi_\phi^n g_{K-Q+2,Q+n-1}(\rho), \tag{63}$$

where

$$\Psi_\phi = |a|^2\mathbf{s}_0^\dagger\mathbf{R}^{-1}\mathbf{s}_0\sin^2\phi, \tag{64}$$

and

$$g_{k,m}(x) = \frac{(k+m-1)!}{(k-1)!(m-1)!}x^{k-1}(1-x)^{m-1} \tag{65}$$

with $0 < x < 1$. According to (48), the detection probability conditioned on ρ in the case of $q=1$ becomes

$$\begin{aligned} P_{\mathrm{D}|\rho} =& 1 - \left[\frac{\rho}{\rho-\xi(\rho+\alpha-\rho\alpha)}\right]^{-(K-Q+1)} \\ &\times \sum_{j=1}^{K-Q+1}\binom{K-Q+1}{j}\left[\frac{\xi(\rho+\alpha-\rho\alpha)}{\rho-\xi(\rho+\alpha-\rho\alpha)}\right]^j \\ &\times \exp\left\{-\delta\left[\rho-\xi(\rho+\alpha-\rho\alpha)\right]\right\} \\ &\times \sum_{m=0}^{j-1}\frac{\delta^m}{m!}\left[\rho-\xi(\rho+\alpha-\rho\alpha)\right]^m, \end{aligned} \tag{66}$$

where the non-centrality parameter δ in the mismatched case becomes

$$\delta = |a|^2\mathbf{s}_0^\dagger\mathbf{R}^{-1}\mathbf{s}_0\cos^2\phi. \tag{67}$$

Therefore, the detection probability of the MRT with $q=1$ can be expressed as

$$P_\mathrm{D} = \int_{\frac{\xi\alpha}{1-\xi+\xi\alpha}}^{1} P_{\mathrm{D}|\rho} f_\rho^{\mathrm{mis}}(\rho)\mathrm{d}\rho, \tag{68}$$

where $P_{\mathrm{D}|\rho}$ and $f_\rho^{\mathrm{mis}}(\rho)$ are given in (66) and (63), respectively.

For the fluctuating target model where $a \sim \mathcal{CN}(0, \sigma_\mathrm{t}^2)$ with σ_t^2 denoting the target power, the detection probability of the MRT can be expressed as

$$P_\mathrm{D} = \int_{\frac{\xi\alpha}{1-\xi+\xi\alpha}}^{1} \underbrace{\int_0^\infty P_{\mathrm{D}|\rho} f_y(y) f_\rho^{\mathrm{mis}}(\rho)\mathrm{d}\delta\,\mathrm{d}\rho}_{U(\rho)}, \tag{69}$$

where $y = |a|^2 \sim \frac{\sigma_\mathrm{t}^2}{2}\chi_2^2$, and the PDF of y is

$$f_y(y) = \frac{1}{\sigma_\mathrm{t}^2}\exp\left(-\frac{y}{\sigma_\mathrm{t}^2}\right). \tag{70}$$

Using (63), (66) and (70), we can obtain

$$U(\rho) = U_1(\rho) - U_2(\rho), \tag{71}$$

where

$$U_1(\rho) = \frac{1}{\sigma_\mathrm{t}^2}\sum_{n=0}^{K-Q+2}\binom{K-Q+2}{n}\frac{K!n!}{(K+n)!} \times \frac{g_{K-Q+2,Q+n-1}(\rho)\eta^n\sin^{2n}\phi}{\left[\eta\rho\sin^2\phi + \sigma_\mathrm{t}^{-2}\right]^{m+n+1}} \tag{72}$$

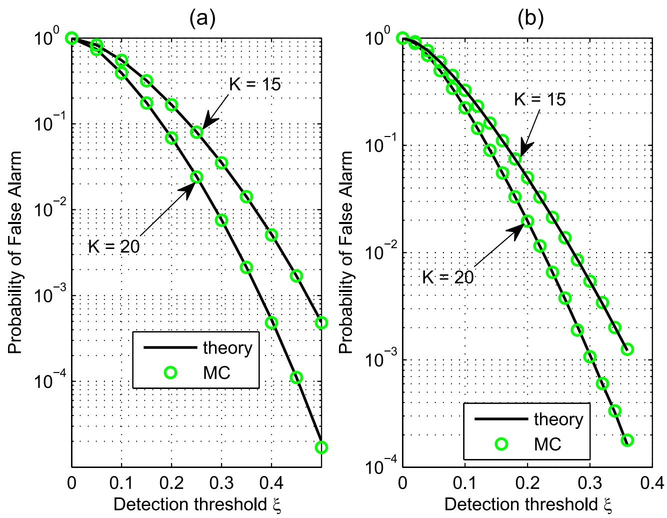

Fig. 1. Probability of false alarm of the MRT versus the detection threshold for $q = 2$. The symbols "○" are MC simulation results, and the lines denote the results obtained with the finite-sum expression in (44).

and

$$U_2(\rho) = \left(\frac{\rho}{\rho-\omega_\rho}\right)^{-(K-Q+1)} \sum_{j=1}^{K-Q+1} \binom{K-Q+1}{j} \times \left(\frac{\omega_\rho}{\rho-\omega_\rho}\right)^{j} \sum_{m=0}^{j-1} \frac{(\eta\cos^2\phi)^m}{m!\sigma_{\rm t}^2}(\rho-\omega_\rho)^m \times \sum_{n=0}^{K-Q+2} \binom{K-Q+2}{n} \frac{K!(m+n)!}{(K+n)!} \times \frac{g_{K-Q+2,Q+n-1}(\rho)\eta^n \sin^{2n}\phi}{\left[\eta(\rho-\omega_\rho)\cos^2\phi + \eta\rho\sin^2\phi + \sigma_{\rm t}^{-2}\right]^{m+n+1}} \tag{73}$$

with $\omega_\rho = \xi(\rho + \alpha - \rho\alpha)$.

It should be pointed out that the detection probability of the MRT is unavailable for the case of $q > 1$, since it is difficult to derive the PDF of ρ for the multi-rank subspace case.

V. Numerical Results

In this section, numerical simulations are conducted to confirm the validity of the above theoretical results. The data model (4) for the polarimetric radar system is used. We choose $M = 5$, $f_{\rm d} = 0.1$, and $\nu = 0.95$. The normalized noise covariance matrix $\mathbf{C}$ is Gaussian shaped with one-lag correlation coefficient 0.9, i.e., $[\mathbf{C}]_{ij} = 0.9^{|i-j|^2}$. Assume that the complex target amplitude vector $\mathbf{a}$ has a circular complex Gaussian distribution, i.e., $\mathbf{a} \sim \mathcal{CN}(\mathbf{0}, \sigma_{\rm t}^2\mathbf{I}_q)$, where $\sigma_{\rm t}^2$ is the target power. Define the SNR in decibel as

$$\mathrm{SNR} = 10\log_{10}\frac{\sigma_{\rm t}^2}{\sigma^2}, \tag{74}$$

where σ^2 is the noise power defined in (6).

The probability of false alarm of the MRT with $q = 2$ as a function of the detection threshold is shown in Fig. 1, where the lines denote the results obtained with the finite-sum expression in (44), and the symbols "○" represent the results obtained with MC simulations. The number of independent trials used in each case is 10^6. It can be seen that the theoretical results are in good accordance with the simulation results.

A. Matched Case

Here, we examine the detection performance of the proposed MRT with $q = 2$ in the matched case where the target signal belongs to the nominal subspace. The probability of false alarm is set to be $P_{\rm FA} = 0.01$ throughout the following simulations.

In Fig. 2, the detection probability of the proposed MRT as a function of SNR is plotted for different K. Here, we choose $\alpha = 1$. Note that the proposed MRT with $\alpha = 1$ corresponds to the Rao test derived in [46] for point-like targets. The solid line denoting the detection probability of the proposed MRT is obtained from the theoretical expression derived in Section IV-B, while the symbols "○" represent the results obtained from MC simulations to provide an independent confirmation of the theoretical results. It is shown that the theoretical results match the MC simulation results pretty well.

For comparison, we consider the AMF (18), the GLRT detector (31), and the ASD given by [45]

$$T_{\rm ASD} = \frac{\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S}\left(\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{S}\right)^{-1}\mathbf{S}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}}{\mathbf{x}^\dagger\hat{\mathbf{R}}^{-1}\mathbf{x}} \underset{H_0}{\overset{H_1}{\gtrless}} t_{\rm ASD}. \tag{75}$$

Note that comparisons between the proposed MRT and the two-stage (or tunable) detectors in [20]–[23], [47] are not made for the case of $q = 2$, since these detectors are designed for $q = 1$, and cannot be applied to the case of $q > 1$.

We can observe in Fig. 2 that when the number of training data is limited (e.g., $K = 12$ in Fig. 2(a) or $K = 15$ in Fig. 2(b)), the detection performance of the Rao test is the worst. This observation is consistent with the analysis in Section III. Interestingly, there exists a ceiling for the detection performance of the Rao test in the case of limited training data. This phenomenon is due to the fact that the estimate of the noise covariance matrix in the Rao test is contaminated by the target signal. Nevertheless, this phenomenon can be alleviated by using sufficient training data (e.g., $K = 20$ in Fig. 2(c)). This is as expected, since the more the secondary data, the less the negative effect of the signal contamination on the noise covariance matrix estimate. When the number of training data is large, the Rao test provides detection performance similar to the counterparts.

Fig. 3 shows the detection probability of the proposed MRT as a function of the tunable parameter α for the case of $q = 2$. It can be seen that as the tunable parameter α increases, the detection probability decreases. Note that $\alpha = 0$ and $\alpha = 1$ correspond to the GLRT detector and the Rao test, respectively. It implies that the proposed MRT provides detection performance no better than the GLRT detector.

B. Mismatched Case

In the above simulations, we assume that there is no mismatch in the signal model. In practice, the signal may not belong to the presumed subspace, due to many factors such as calibration errors, and wavefront distortions. We first consider the mismatched case of $q = 2$ where the actual signal subspace denoted by $\mathbf{S}_0$ is not aligned with the nominal subspace $\mathbf{S}$. The amount of mismatch between $\mathbf{S}_0$ and $\mathbf{S}$ is measured by (61).

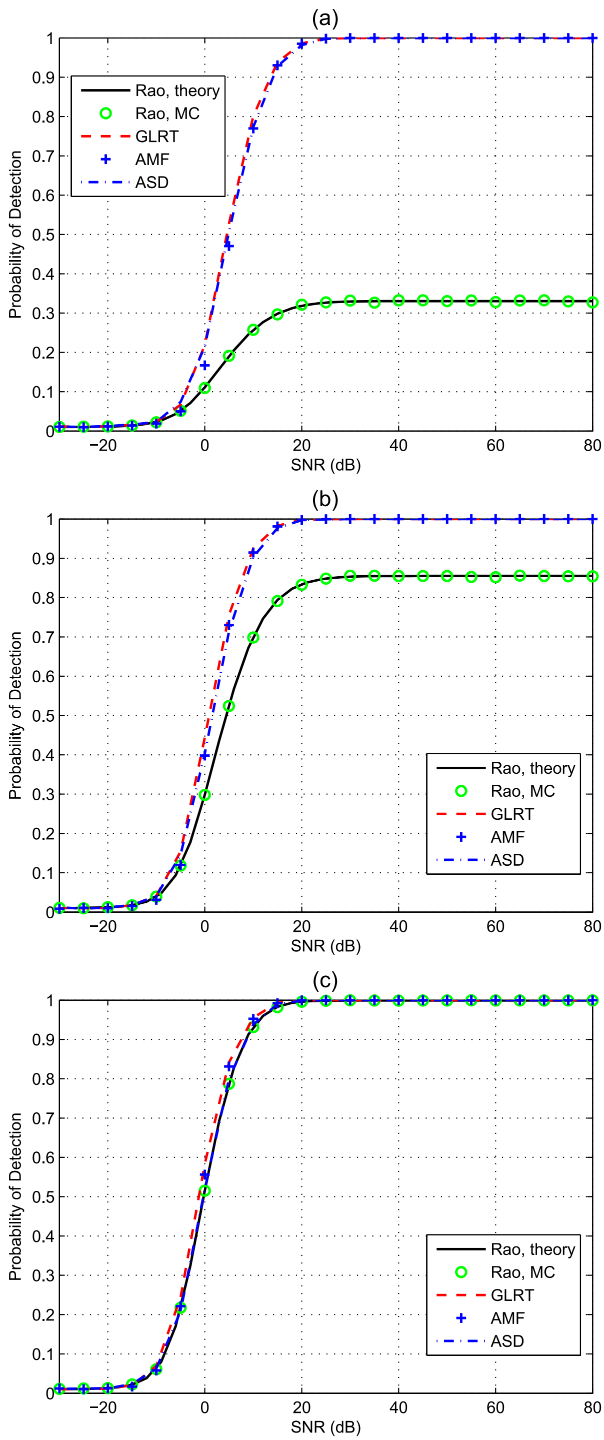

Fig. 2. Detection probability versus SNR with $q = 2$, $Q = 10$, and $\alpha = 1$ in the matched case. The symbols "○" denote the results obtained by MC simulations, and the lines denote the results obtained with the theoretical expression in (59). (a) K = 12; (b) K = 15; (c) K = 20.

In Fig. 4, the detection probability curves of the MRT with $q = 2$ with different tunable parameters are plotted with respect to $\cos^2\phi$ for SNR = 10 and 15 dB. All the curves are obtained with MC simulations. We can observe that the selectivity of the proposed MRT can be flexibly controlled by adjusting the tunable parameter α. Specifically, the rejection capabilities of mismatched signals of the MRT increase as the tunable parameter α increases.

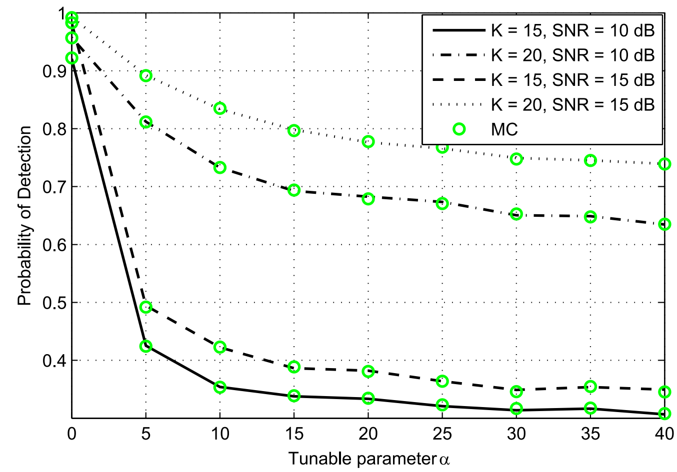

Fig. 3. Detection probability versus the tunable parameter α for $q = 2$ in the matched case. The symbols "○" denote the results obtained by MC simulations, and the lines denote the results obtained with the theoretical expression in (59).

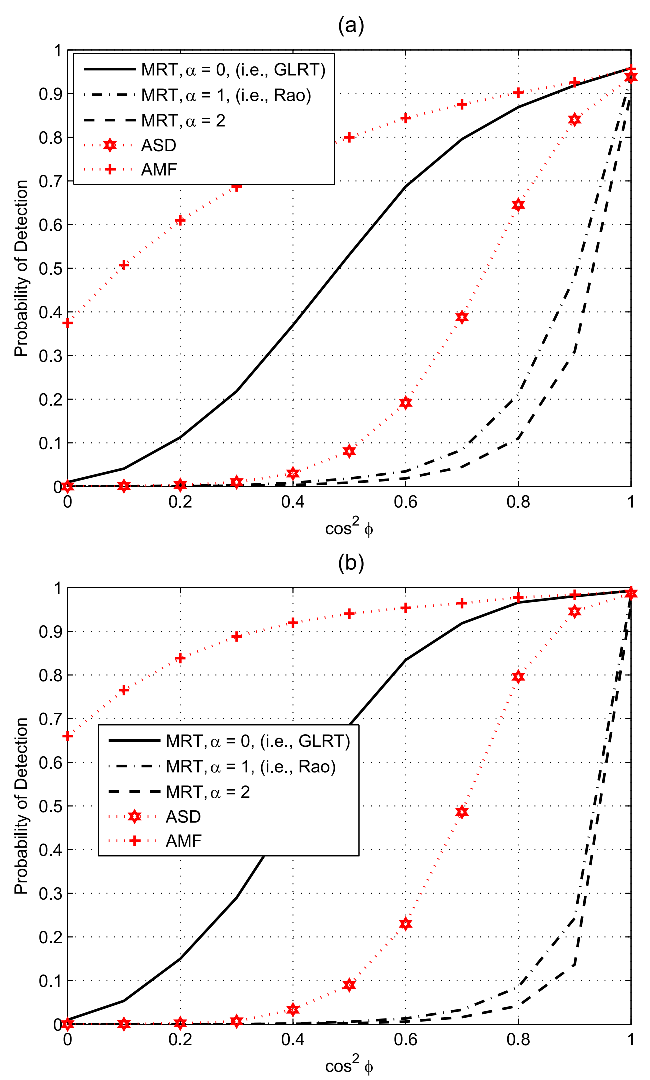

Fig. 4. Detection probability versus $\cos^2\phi$ for $q = 2$ and $K = 20$ in the mismatched case. (a) SNR = 10 dB; (b) SNR = 15 dB

Interestingly, the MRT with $\alpha = 1$ (i.e., the Rao test) has better rejection of mismatched signals than the ASD for the considered scenario. It is also shown that when the tunable parameter α increases from 0 to 1, the selectivity of the MRT dramatically improves. Further increase in the tunable parameter appears to lead to a slight improvement in the capability of rejecting mismatched signals. In addition, it is observed that the mismatched rejection capability of the AMF is the worst, which was also observed in [11], [15].

Next, we examine the mismatched case of $q = 1$, where the subspace matrix $\mathbf{S}$ reduces to a steering vector denoted by $\mathbf{s}$. Fig. 5 shows the contour of constant detection probability of the MRT with respect to $\cos^2\phi$ for $q = 1$. Note that the existing two-stage detectors and tunable detectors can be applied to the case of $q = 1$. For comparison purposes, we consider two tunable detectors (i.e., the KWA [23] and the tunable detector in [47]) and three two-stage detectors (i.e., the WAS-ASB [22], S-ASB [21], and the AMF-GLRT detector [20]). In the WAS-ASB and S-ASB, the subspace $\mathbf{H}$ used in the first stage for taking into account the uncertainty in the signal steering vector is chosen as

$$\mathbf{H} = [\mathbf{s}^T, \breve{\mathbf{s}}^T]^T, \tag{76}$$

where $\mathbf{s}$ is defined in (3), and

$$\breve{\mathbf{s}} = \frac{1}{\sqrt{M}}[1, \cdots, \exp(\jmath 2\pi(M-1)(f_{\rm d}+0.01))]^T. \tag{77}$$

For fair comparisons, the threshold pair for each two-stage detector (or the tunable parameter in each tunable detector) is chosen to ensure about 1 dB SNR loss with respect to Kelly's GLRT detector for matched signals at $P_D = 0.9$. It can be seen that in this parameter setting the selectivity of the proposed MRT is slightly better than the KWA and WAS-ASB at the region of high SNR, and significantly better than the other detectors.

Note that all the curves in Fig. 5 are obtained by MC simulations. We also used the closed-form expression in (69) to calculate the contour of the constant detection probability of the MRT, and found that the theoretical results are consistent with the MC ones. For clarity of exposition, the theoretical results are not presented in Fig. 5.

VI. Conclusion

In this paper, we examined the multi-rank subspace signal detection problem. A simple derivation of the Rao test was presented by using both the test and training data in the noise correlation estimation. Moreover, we proposed the MRT by introducing a tunable parameter. It subsumes the GLRT and Rao test as particular cases. The performance of the proposed MRT is evaluated in terms of the probabilities of false alarm and detection. It is shown that the MRT has the CFAR property with respect to the noise covariance matrix. In practice, the detection threshold of the MRT can be easily set by using the closed-form expression for the probability of false alarm. Simulation results reveal that the mismatched signal rejection capabilities of the proposed MRT can be flexibly adjusted. Specifically, the mismatched signal rejection capabilities improve as the tunable parameter increases. When the tunable parameter is sufficiently large, the rejection performance of the proposed MRT is better than that of its counterparts.

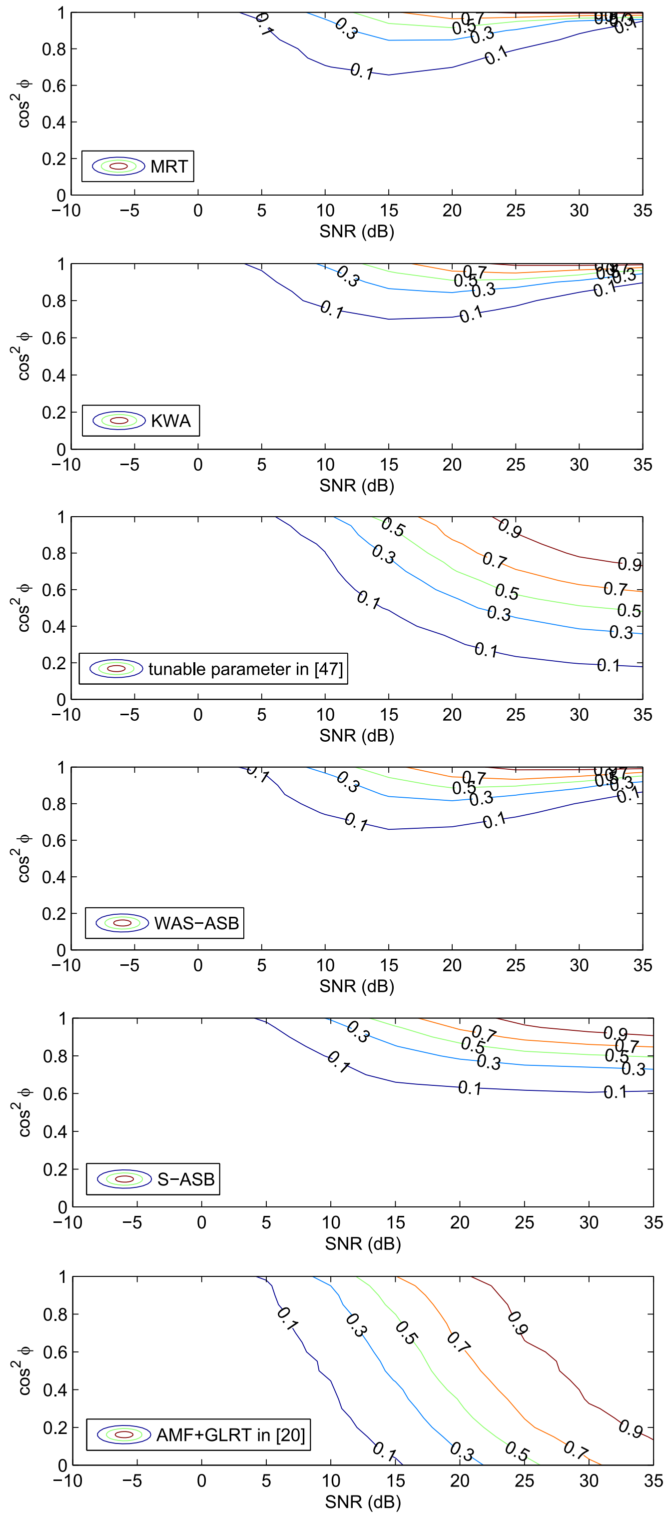

Fig. 5. Contour of constant detection probability for $q = 1$ and $K = 10$ in the mismatched case.

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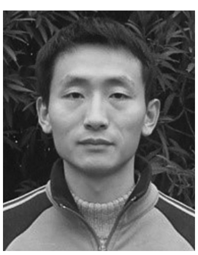

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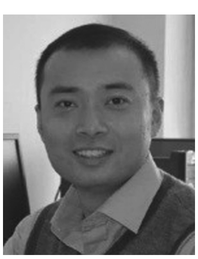

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