GROUND SURFACE VIBRATION DUE TO AXISYMMETRIC WAVE MOTION IN BURIED FLUID-FILLED PIPES

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This paper is concerned with the ground surface vibration induced by the axisymmetric fluid-borne wave motion in buried fluid-filled pipes. This wave has been exploited for water leak detection and, more recently, for the detection and location of buried pipes. Based on the model of wave propagation developed in recent work, this paper presents an analytical method for predicting the ground surface displacement resulting from the radiated elastic waves in the soil medium. It is shown that, for a sufficient large wavenumber-depth product, the radiated conical compressional and shear waves can be treated as plane waves incident upon the ground surface. Analytical expressions for the ground surface displacement are then given as the overall contributions from these incident waves and their reflections. Numerical simulations are further presented to predict the relative displacements normal and parallel to the ground surface under two representative pipe-soil boundary conditions, representing "perfect bonding" and "imperfect bonding". Theoretical predictions may help to explain some of the features observed in practice and hence offer a potential improvement over the current acoustic techniques for leak detection and pipe location.

1. Introduction

The study of acoustic wave propagation in buried fluid-filled pipes has received much attention in the past decade. Earlier research has shown that at low frequencies (well below the pipe ring frequency), the axisymmetric (n=0) fluid-borne (s=1) wave, which propagates at low frequencies in fluid-filled pipes, is often the main carrier of vibrational energy within piping systems [1, 2]. Pinnington and Briscoe [2] developed a theoretical model of axisymmetric wavenumbers in fluid-filled pipes *in vacuo*. Muggleton *et al*. [3, 4] focused on the wavenumber predictions of axisymmeric waves in buried fluid-filled pipes, and conducted some experiments to validate the predictions for the s=1 wave. This work was extended to consider the 3D effect of the soil on the wavenumber predictions under a lubricated contact condition [5]. More recently, based on the characteristic equation for the cylindrical tri-layer system, an improved model of axisymmetric waves has been developed by the present authors [6] in order to better understand wave propagation within buried fluid-filled pipes and the energy leakage into the surrounding soil. The theoretical model for the s=1 wavenumber predictions have been validated by some experiments conducted on MDPE water pipes surrounded by an air, fluid and soil media. Based on the knowledge of the characteristics of the propagation s=1 wave, substantial research has been conducted by the present authors for water leak detection in buried plastic pipes [7-10] and for the detection and location of buried pipes [11]. Recent work has demonstrated that measuring the ground vibration response resulting from intentionally exciting a pipe can be successful for detecting both cast iron and plastic water pipes in a variety of soils. However, to date, modelling work has been limited to calculating the phase velocity and attenuation of the s=1 wave; whilst the expected behaviour at the ground surface can be inferred from this in general terms, no analytical model has previously existed for describing the ground surface response in detail.

In this paper, a method is presented for predicting the ground surface displacement resulting from the s=1 wave motion in a buried fluid-filled pipe. For a sufficient large wavenumber-depth product, the radiated conical compressional and shear waves can be considered to be in the far field at the ground surface, and then each is treated as a plane wave incident upon it. A model of wave propagation developed in recent work is incorporated into the analytical method to predict the ground surface displacements due to the radiated elastic waves in the soil medium. Analytical solutions for the ground surface displacement are then given as the sum of the contributions from these incident waves and their reflections. Numerical results are presented for the prediction of the relative displacements at the ground surface under two pipe-soil boundary conditions, representing "imperfect bonding" and "perfect bonding", for a PVC water pipe buried in sandy soil. The theoretical work with numerical analyses provide the basis for better understanding the "phase shift" phenomenon in practical measurements [11] and predicting the ground surface vibration for pipe location using acoustic techniques.

2. Radiation in the soil

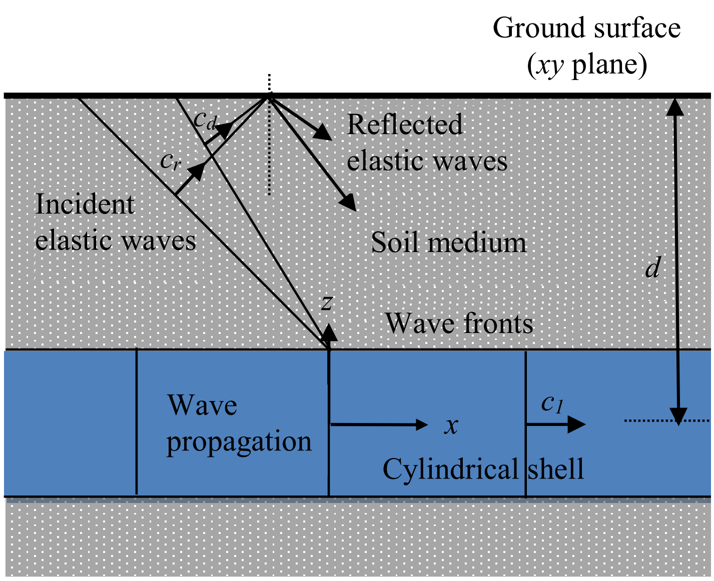

Figure 1. Illustration of reflection of the radiated elastic waves induced by the axisymmetric mode in the Cartesian coordinate (x, y, z) system. c_1, c_r and c_d denote the phase velocity of the s=1 wave, compressional and shear velocities in the soil respectively.

Whilst an infinite elastic medium is useful for gaining an understanding of the characteristics of wave propagation in the buried fluid-filled pipe, further development of the theory is necessary to determine the ground surface motion due to radiation in the soil. Recent investigation [6] has shown that the fluid-borne wave, in general, will leak shear waves for typical values of the shear velocity in the soil ($c_1 > c_r$), while may or may not leak compressional waves into the soil. For example, the compressional wave will radiate in some loose, sandy soils (where, in general, $c_1 > c_d$), but will be less likely to radiate in clay soils (where, typically, $c_1 < c_d$). A general model is developed when both elastic waves radiate throughout the analysis.

For soil vibration, the compressional and shear wave potentials in the cylindrical coordinates (r, θ, x) have the form $\phi_m = A_m \mathrm{H}_0(k_d^r r)\mathrm{e}^{\mathrm{i}(\omega t - k_1 x)}$ and $\psi_m = B_m \mathrm{H}_0(k_r^r r)\mathrm{e}^{\mathrm{i}(\omega t - k_1 x)}$ respectively, where A_m and B_m are potential coefficients, which are found to depend upon the pipe-soil boundary conditions, and thereby can be expressed in terms of the pipe wall displacements [6]; $\mathrm{H}_0(\,)$ is the Hankel function of the second kind and zero order representing an outgoing wave; k_d^r and k_r^r, are the radial components of the compressional and shear wavenumbers, k_d and k_r, in the surrounding medium respectively, $(k_d^r)^2 = k_d^2 - k_1^2$ and $(k_r^r)^2 = k_r^2 - k_1^2$. For large arguments (i.e., wavenumber-depth products, kr) of the Hankel functions $|kr| > 1$, the Hankel functions can be approximated by the asymptotic expansions, $\mathrm{H}_\gamma(kr) \doteq \sqrt{2/\pi kr} e^{-\mathrm{i}(kr - \gamma\pi/2 - \pi/4)}$ for an integral γ. These approximations are adopted to allow the radiated conical elastic waves to be treated as plane ways as they reach the ground surface. The cylindrical coordinate system is then transformed into a Cartesian coordinate system more appropriate to the analysis of the plane waves. A similar approach was taken by Jette and Parker in [12].

With reference to Fig. 1, the radiated elastic waves travelling over a depth d (note that the distance d is calculated from the pipe centre) can be treated as plane waves at the ground surface provided that $|k_d^r d| > 1$ and $|k_r^r d| > 1$. In addition, only excitation of the ground directly over the pipe is considered. Analytical description of the interaction of cylindrical or conical waves with a planar surface is complex. For example, at a lateral distance from the pipe axis comparable to the pipe depth, an incident conical shear wave can excite surface waves of various kinds in addition to the reflected bulk waves [13, 14]. Directly over the pipe, which is the main region of interest for this study, surface waves do not develop, and only reflected elastic waves need to be considered.

Provided this limitation is recalled, the resultant field can be considered to be independent of y; the vertical (z direction) and horizontal (x direction) soil displacements in the Cartesian coordinates can then be expressed in terms of two wave potentials by

(1) $$u_x = \frac{\partial \phi_p}{\partial x} + \frac{\partial \psi_p}{\partial z}\,;\; u_z = \frac{\partial \phi_p}{\partial z} - \frac{\partial \psi_p}{\partial x}$$

where the compressional and shear potentials $\phi_p = \Phi_p \mathrm{e}^{\mathrm{i}(\omega t - k_d^r z - k_1 x)}$ and $\psi_p = \Psi_p \mathrm{e}^{\mathrm{i}(\omega t - k_r^r z - k_1 x)}$ respectively; Φ_p and Ψ_p are the potential amplitudes, with the subscript p denoting plane waves. To set up the relationship between the two coordinate systems, it is assumed that at the ground surface (z=d), the vertical and horizontal soil displacements given by Eq. (1) are equivalent to the corresponding soil displacements in the cylindrical coordinates (r=d). Adopting the far field asymptotic approximations for the Hankel functions gives

(2) $$\Phi_p = A\sqrt{2/\pi k_d^r d}\, e^{\mathrm{i}\pi/4}\,,\; \Psi_p = \mathrm{i}B\sqrt{2k_r^r/\pi d}\, e^{\mathrm{i}\pi/4}$$

When radiated waves are incident upon the ground surface, reflections occur as illustrated in Fig. 1. The potentials ϕ_p and ψ_p can be expressed as overall contributions of the incident and reflected wave potentials by

(3) $$\phi_p = (\Phi_p^+ \mathrm{e}^{-\mathrm{i}k_d^r z} + \Phi_p^- \mathrm{e}^{\mathrm{i}k_d^r z})\mathrm{e}^{\mathrm{i}(\omega t - k_1 x)}\ ;\ \psi_p = (\Psi_p^+ \mathrm{e}^{-\mathrm{i}k_r^r z} + \Psi_p^- \mathrm{e}^{\mathrm{i}k_r^r z})\mathrm{e}^{\mathrm{i}(\omega t - k_1 x)}$$

where Φ_p^+ and Ψ_p^+ are the incident potential amplitudes; Φ_p^- and Ψ_p^- are the reflected potential amplitudes. The normal and tangential stresses in the surrounding medium are given by

(4) $$\sigma_{zx} = \mu_m \left(2\frac{\partial^2 \phi_p}{\partial x \partial z} - \frac{\partial^2 \psi_p}{\partial x^2} + \frac{\partial^2 \psi_p}{\partial z^2} \right);\ \sigma_{zz} = (\lambda_m + 2\mu_m)\frac{\partial^2 \phi_p}{\partial z^2} + \lambda_m \frac{\partial^2 \phi_p}{\partial x^2} - 2\mu_m \frac{\partial^2 \psi_p}{\partial x \partial z}$$

where ρ_m, λ_m and μ_m are the density and Lamé coefficients of the surrounding soil.

The potential amplitudes of the reflected waves can be determined from the boundary conditions at the ground surface, where both normal and tangential stresses vanish. Substituting Eqs. (3) into (4) and setting $\sigma_{zx} = 0$ and $\sigma_{zz} = 0$ at z=d gives

(5) $$\begin{pmatrix} \Phi_p^- \\ \Psi_p^- \end{pmatrix} = \mathbf{R} \begin{pmatrix} \Phi_p^+ \\ \Psi_p^+ \end{pmatrix}$$

where the elements R_{ij} are given by

(6) $$R_{11} = C_{11}\mathrm{e}^{-2\mathrm{i}k_d^r d}\ ;\ R_{12} = C_{12}\mathrm{e}^{-\mathrm{i}(k_d^r + k_r^r)d}\ ;\ R_{21} = C_{21}\mathrm{e}^{-\mathrm{i}(k_d^r + k_r^r)d}\ ;\ R_{22} = C_{11}\mathrm{e}^{-2\mathrm{i}k_r^r d}$$

where $C_{11} = \dfrac{4k_1^2 k_d^r k_r^r - \left[(k_r^r)^2 - k_1^2\right]^2}{4k_1^2 k_d^r k_r^r + \left[(k_r^r)^2 - k_1^2\right]^2}$; $C_{12} = \dfrac{4k_1 k_r^r \left[(k_r^r)^2 - k_1^2\right]}{4k_1^2 k_d^r k_r^r + \left[(k_r^r)^2 - k_1^2\right]^2}$; $C_{21} = -\dfrac{4k_1 k_d^r \left[(k_r^r)^2 - k_1^2\right]}{4k_1^2 k_d^r k_r^r + \left[(k_r^r)^2 - k_1^2\right]^2}$.

Combining Eqs. (1-3) and (5) results in the soil displacements at the ground surface in terms of the constants A and B by

(7) $$\begin{pmatrix} u_x \\ u_r \end{pmatrix} = \sqrt{\frac{2}{\pi d}}\mathrm{e}^{\mathrm{i}\pi/4} \begin{bmatrix} -\mathrm{i}\left(1 + C_{11} - \dfrac{k_r^r}{k_1}C_{21}\right)k_1 (k_d^r)^{1/2}\mathrm{e}^{-\mathrm{i}k_d^r d} & \left(1 - C_{11} + \dfrac{k_1}{k_r^r}C_{12}\right)(k_r^r)^{3/2}\mathrm{e}^{-\mathrm{i}k_r^r d} \\ -\mathrm{i}\left(1 - C_{11} - \dfrac{k_1}{k_d^r}C_{21}\right)(k_d^r)^{1/2}\mathrm{e}^{-\mathrm{i}k_d^r d} & -\left(1 + C_{11} + \dfrac{k_d^r}{k_1}C_{12}\right)k_1 (k_r^r)^{1/2}\mathrm{e}^{-\mathrm{i}k_r^r d} \end{bmatrix} \begin{pmatrix} A \\ B \end{pmatrix} \mathrm{e}^{\mathrm{i}(\omega t - k_1 x)}$$

where the constants A and B can be determined from the boundary conditions at the pipe-soil interface. The details have been discussed in [6], and therefore not repeated here.

3. Numerical results and discussions

This section presents some numerical results of the ground surface displacements relative to the pipe wall displacements induced by the s=1 wave motion in a water-filled PVC pipe (of diameter 169mm and thickness 11mm) buried in sandy soil (d=1.0845m). The real and imaginary parts of the wavenumber are also plotted to show the dispersive nature of the s=1 wave. Losses within the pipe wall are included (with a loss factor 0.065) and the surrounding medium is assumed to be lossless. Material properties of the pipe, soil and fluid are shown in Table 1. The frequency range of interest is up to 1kHz, since signals are heavily attenuated at higher frequencies in plastic pipes [6].

Table 1. Material properties for theoretical predictions

(ρ-density; c_l, c_d-longitudinal velocity; c_r-shear velocity)

Material	PVC	Soil	Water
ρ (kg/m^3)	2000	2000	1000
c_l, c_d (m/s)	1725	200	1500
c_r (m/s)	-	100	0

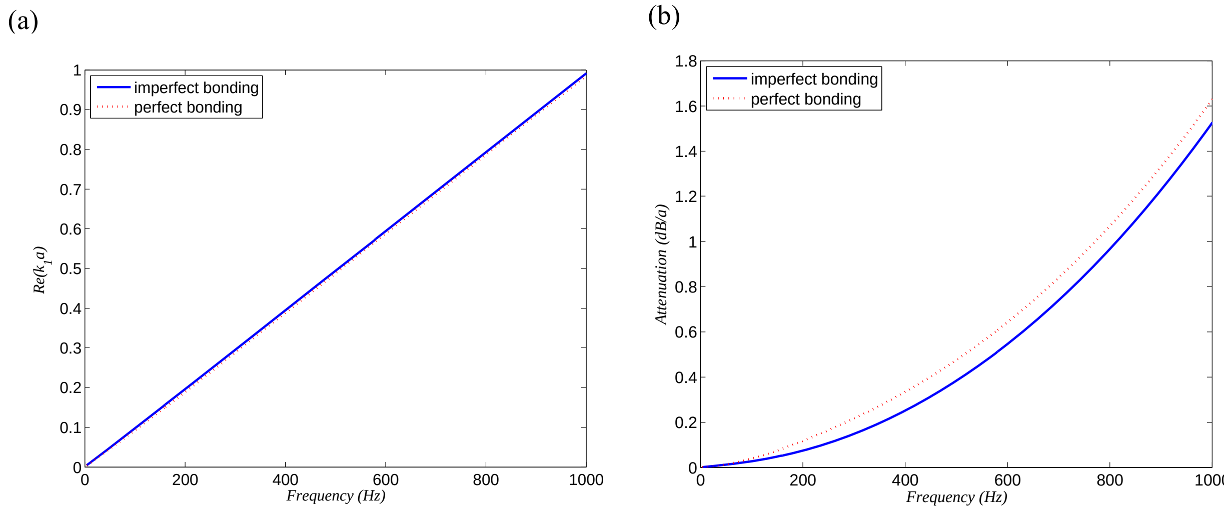

Figure 2. Dispersion curves for the s=1 wave in a PVC water pipe buried in sandy soil: (a) real part of the wavenumber; (b) attenuation. The results are non-dimensionalised by the pipe radius a.

It can be seen from Fig. 2(a) that the difference in the real part of the wavenumber (relating to the phase velocity) between two boundary conditions is marginal in sandy soil. A slight decrease of the real part of the wavenumber for “perfect bonding” is due to the increase of the pipe wall stiffness caused by the presence of shear at the pipe-soil interface. Fig. 2(b) shows that the attenuation increases under “perfect bonding” compared with “imperfect bonding”, more details of which can be found in [6]. This confirms that the presence of shear at the pipe-soil interface increases the attenuation due to the added radiation damping effect, except at very low frequencies when losses within the pipe wall dominate. The calculated phase velocity of the s=1 wave is greater than both the compressional and shear velocities in the soil. Therefore both elastic waves will radiate in the sandy soil considered.

The ground surface displacement are evaluated in term of the amplification factor, which is defined as the ratio of the displacement amplitudes at the ground surface and the pipe wall in the same direction. For the buried PVC pipe considered in this analysis, it is found that the two Hankel function arguments satisfy $\left|k_d^r d\right| > 1$ above 34Hz and $\left|k_r^r d\right| > 1$ above 17Hz. Fig. 3(a) shows the magnitude of the vertical amplification factor. It can be seen that the ground surface displacement is amplified below 200Hz under “imperfect bonding” conditions. It decreases gradually with frequency and approaches 0.5 at higher frequencies. The unwrapped phase of the vertical amplification factor decreases linearly with frequencies, as illustrated in Fig. 3(b), with no abrupt changes. For “perfect bonding”, it can be observed from Fig. 3(a) that the magnitude of the amplification factor also decreases with frequency with some fluctuations, which are distinctive at lower frequencies. The magnitude reaches the minimum value (approximately zero). In this case, although the unwrapped phase decreases linearly for the most part, an additional phase jump of around π occurs at 145Hz. At this point, the cause of this behaviour is not completely clear but it could result from the inference of incident and reflected radiated waves at the ground surface as also suggested by Jette and Parker [12].

The magnitude and phase of the horizontal amplification factor are plotted in Figs. 4(a) and (b) respectively. Comparison of the magnitudes of the amplification factor shows different trends under both boundary conditions. For "imperfect bonding", the magnitude fluctuates between 0 and 0.6 separated by approximately 190Hz due to the interaction of the incident and reflected radiated waves. Correspondingly, although the unwrapped phase generally decreases linearly with frequency, additional phase shifts occur at the frequencies when the magnitude approaches a local minimum (approximately zero), as shown in Figs. 4(b). When the shear stress is accounted for at the interface ("perfect bonding"), a similar trend is observed in the magnitude and phase of the vertical amplification factor compared to the horizontal factor. Here however, fluctuations in the magnitude are more obvious at higher frequencies as illustrated in Figs. 4(a) rather than at lower frequencies for the vertical displacements. The magnitude reaches approximately zero and phase shifts occur at 810Hz.

(a)

(b)

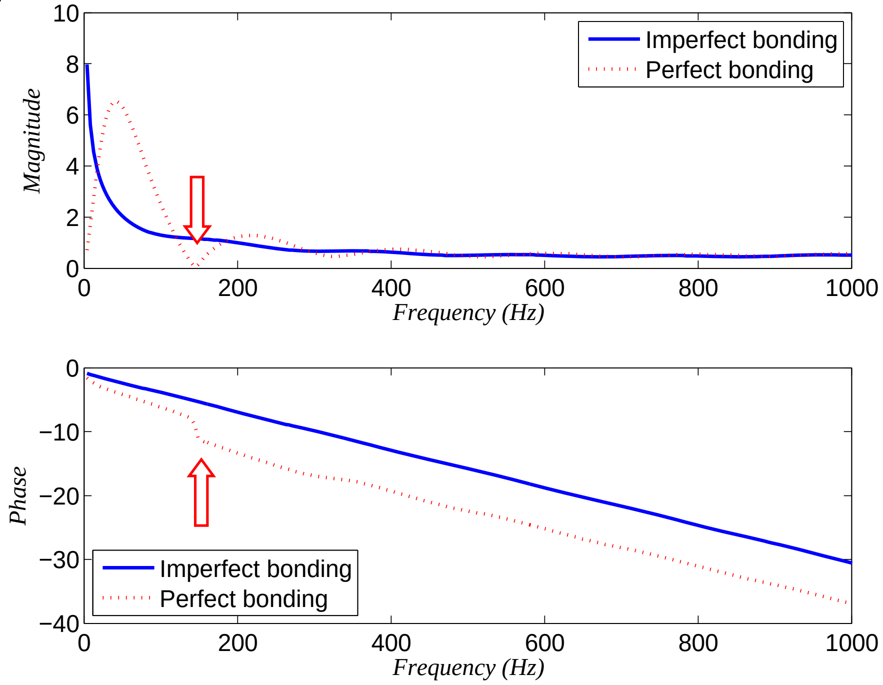

Figure 3. The vertical amplification factor: (a) magnitude; (b) unwrapped phase. The additional phase shifts are marked by the red arrow for "perfect bonding".

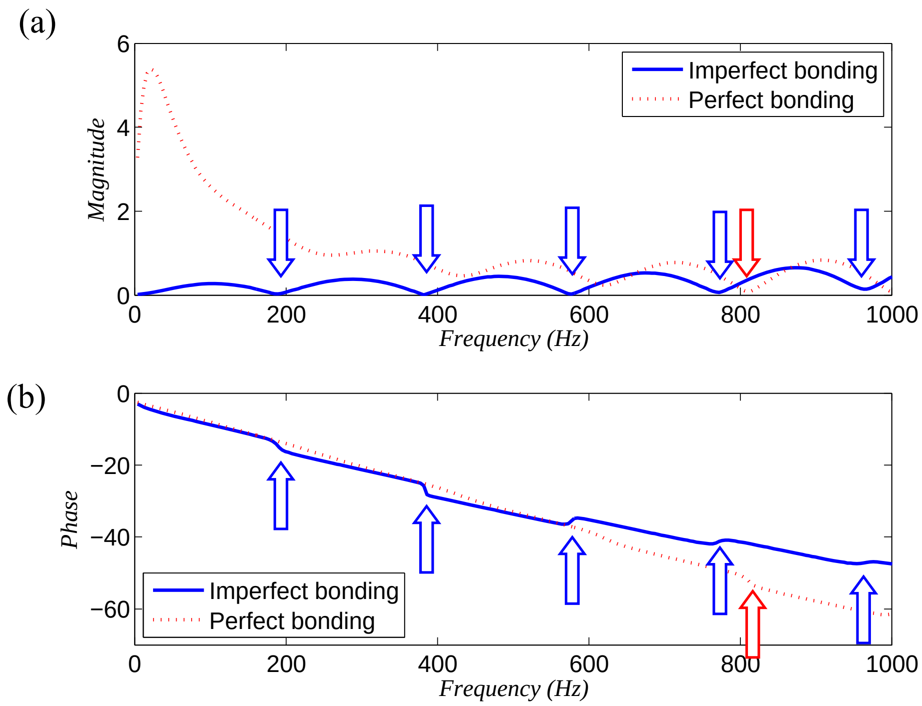

Figure 4. The horizontal amplification factor: (a) magnitude; (b) phase. The additional phase shifts are marked by the blue arrows for "imperfect bonding" and the red arrow for "perfect bonding".

4. Conclusions

Based on a model of axisymmetric wave propagation developed previously, the ground surface displacements due to waves radiating from a buried fluid-filled pipe has been studied in this paper. Above a certain frequency (i.e., for a sufficient large wavenumber-depth product), the radiated conical compressional and shear waves can be treated as the far-field planes waves at the ground surface, whereby the relative displacements at the ground surface can be determined. Numerical examples have been presented to predict the ground vibration displacements under two boundary conditions at the pipe-soil interface. Theoretical predictions have shown that, although a general decrease with frequency in the magnitude of the displacement amplification factors is observed, fluctuations also occur. The unwrapped phase of the amplitude factors generally decreases linearly but additional phase jumps are seen. These phase shifts occur at the local minima (approximately zero) of the amplitude factor magnitudes. It seems likely that these phenomena occur as a result of inference between the incident and reflected radiated waves but this has not been confirmed definitively in this study. Future work will address this issue more thoroughly. On a practical note, when attempting to remotely determine the run of a buried pipe using the pipe excitation method described in [11], examination of the fluctuations in the magnitude of the ground surface response may provide the additional information required to estimate the pipe's depth.

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REFERENCES

1. Fuller, C. R. and Fahy, F. J. Characteristics of wave-propagation and energy-distributions in cylindrical elastic shells filled with fluid, *Journal of Sound and Vibration,* **81**, 501-518, (1982).
2. Pinnington, R. J. and Briscoe, A. R. Externally applied sensor for axisymmetrical waves in a fluid-filled pipe, *Journal of Sound and Vibration,* **173**, 503-516, (1994).
3. Muggleton, J. M., Brennan, M. J. and Pinnington, R. J. Wavenumber prediction of waves in buried pipes for water leak detection, *Journal of Sound and Vibration* 249 (2002) 939-954.
4. Muggleton, J. M., Brennan, M. J. and Linford, P. W. Axisymmetric wave propagation in fluid-filled pipes: wavenumber measurements in in vacuo and buried pipes, *Journal of Sound and Vibration* 270 (2004)171-190.
5. Muggleton, J. M. and Yan, J. Wavenumber prediction and measurement of axisymmetric waves in buried fluid-filled pipes: Inclusion of shear coupling at a lubricated pipe/soil interface, *Journal of Sound and Vibration* 332 (2013) 1216-1230.
6. Gao, Y., Muggleton, J. M., Yang, J. and Tian, J. Theoretical and experimental investigation into axisymmetric quasi-longitudinal waves in thin-walled fluid-filled pipes surrounded by an elastic medium, *submitted to Journal of Sound and Vibration,* (2015).
7. Gao, Y., Brennan, M. J., Joseph, P. F., Muggleton, J. M. and Hunaidi, O. A model of the correlation function of leak noise in buried plastic pipes, *Journal of Sound and Vibration,* **277,** 133-148, (2004).
8. Gao, Y., Brennan, M. J., Joseph, P. F., Muggleton, J. M. and Hunaidi, O. On the selection of acoustic/vibration sensors for leak detection in plastic water pipes, *Journal of Sound and Vibration,* **283** (3-5), 927-941, (2005).
9. Gao, Y., Brennan, M. J. and Joseph, P. F. A comparison of time delay estimators for the detection of leak noise signals in plastic water distribution pipes, *Journal of Sound and Vibration,* **292,** 552-570, (2006).
10. Brennan, M. J., Gao, Y. and Joseph, P. F. On the relationship between time and frequency domain methods in time delay estimation for leak detection in water distribution pipes, *Journal of Sound and Vibration* **304**, 213-223, (2007).
11. Muggleton, J. M., Brennan, M. J. and Gao, Y. Determining the location of buried plastic water pipes from measurements of ground surface vibration, *Journal of Applied Geophysics,* **75,** 54-61, (2011).
12. Jette, A. N. and Parker, J. G. Surface displacment accompanying the propagation of acoustic waves within an underground pipe, *Journal of Sound and Vibration*, **69** (2), 265-274, (1980).
13. Ewing, W. M., Jardetzky, W. S. and Press, F. Elastic waves in layered media, New York: McGraw-Hill, (1957).
14. Jew, A. N. and Parker, J. G. Excitation of an elastic half-space by a buried line source of conical waves, *Journal of Sound and Vibration*, **67**, 523-531 (1979).