An auto-calibration algorithm for uniform circular array with unknown mutual coupling

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***Abstract*—A new subspace-based auto-calibration algorithm for uniform circular array with unknown mutual coupling is presented in this letter. In allusion to the existing ambiguity problems and the limitation of nonzero coupling coefficients in [16], a more generalized iterative method is proposed to jointly estimate the direction-of-arrival (DOA) and unknown mutual coupling. It suffers from no ambiguity problems and does not require the prior knowledge of the number of nonzero elements in mutual coupling vector. Simulation results show the robustness, effectiveness and higher estimated accuracy of the proposed algorithm.**

***Index Terms*—Array mutual coupling, auto-calibration, direction-of-arrival (DOA) estimation, uniform circular array.**

I. Introduction

THe problem of estimating the direction-of-arrival (DOA) of multiple narrowband signals impinging on an array has attracted considerable attention in the last decades [1]. In particular, a variety of high-resolution algorithms which exhibit potentially excellent performance have been proposed [2], [3]. Most of these algorithms assume the array manifold is perfectly known. However, in practice, the array manifold is often affected by unknown array characteristics such as mutual coupling [4], [5], which can seriously degrade the high-resolution algorithms' performance [6], [7].

To solve the problem of array mutual coupling, many calibration algorithms have been published in literature [9]–[17]. The algorithms of [9]–[12] are categorized as offline calibration method since all of them require calibration sources. However, this kind of calibration method has the drawbacks of being expensive and time consuming. Moreover, the additional calibration sources are sometimes not available in practice. Therefore, another way of array calibration, which is the so-called online calibration or auto-calibration, has aroused much interest these years [13]–[17]. These auto-calibration methods are more attractive because they can estimate the DOAs and the unknown mutual coupling coefficients simultaneously without any calibration sources. In [14], a novel online calibration algorithm to compensate for mutual coupling in uniform linear array (ULA) is developed. [15] presents a MUSIC-based 2-D DOA estimation algorithm in the presence of unknown mutual coupling for uniform rectangular array. However, neither of them have considered the mutual coupling calibration for uniform circular array (UCA).

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In contrast to the widely applied ULA, UCA can form uniform beams over 360° azimuthal directions and has been used in more and more applications [8]. Unfortunately, the mutual coupling effect of a UCA can be much stronger than that of a ULA. So it's of great necessity to develop calibration techniques for mutual coupling in UCA. [16] and [17] have proposed two self-calibration algorithms for UCA. They can simultaneously estimate the DOAs and coupling coefficients by using the special structure of mutual coupling matrix (MCM). However, both of them have treated the number of nonzero coefficients in mutual coupling vector as a prior knowledge, which is usually not known exactly for a practical UCA because of the changing environment. More importantly, there exist serious ambiguity problems during the estimation process in [16] and [17], which has been analysed in [18] and [19]. [19] presents a method to estimate unknown mutual coupling and DOAs in beam space, while it should also know the number of nonzero mutual coupling coefficients previously.

In order to overcome the drawbacks above, an iterative auto-calibration algorithm for unknown mutual coupling in UCA is presented in this letter. It is based on subspace theory and utilizes the complex symmetric circular Toeplitz structure of MCM as well. While it does not require any prior knowledge of mutual coupling coefficients and can solve the ambiguity problems very well. Moreover, it is proved to exhibit higher estimated accuracy.

II. Problem formulation

Consider K uncorrelated narrow-band signals impinging on an M-sensor uniform circular array (UCA) of radius r lying on the xy-plane. The impinging signals and the UCA are assumed to be coplanar. Ideally, the vector of M sensor outputs can be written as

$$\mathbf{x}(t) = \sum_{k=1}^{K} \mathbf{a}(\varphi_k) s_k(t) + \mathbf{n}(t) \in \mathbb{C}^{M,1}, \quad (1)$$

where $\mathbf{a}(\varphi_k) = \left[e^{j\beta r\cos(\varphi_k-\phi_1)}, e^{j\beta r\cos(\varphi_k-\phi_2)}, \dots, e^{j\beta r\cos(\varphi_k-\phi_M)}\right]^T \in \mathbb{C}^{M,1}$ is the ideal steering vector for the kth signal $s_k(t)$ impinging from direction φ_k. Here, $\beta = 2\pi/\lambda$ is the wave number and $\phi_m = (m-1)\frac{2\pi}{M}$ $(m = 1, \cdots, M)$ is the azimuth angle of the mth sensor. $\mathbf{n}(t)$ is the vector of additive noise.

Using matrix notation, (1) can be rewritten as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \in \mathbb{C}^{M,1}, \quad (2)$$

where $\mathbf{A} = [\mathbf{a}(\varphi_1), \mathbf{a}(\varphi_2), \dots, \mathbf{a}(\varphi_K)] \in \mathbb{C}^{M,K}$ and $\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_K(t)]^T \in \mathbb{C}^{K,1}$ denote the array manifold matrix and the impinging signal vector, respectively.

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Equation (2) presents the ideal array signal vector model. However, in the presence of mutual coupling, the array steering vector for an arbitrary angle φ should be modified as

$$\tilde{\mathbf{a}}(\varphi) = \mathbf{C}\mathbf{a}(\varphi) \in \mathbb{C}^{M,1}, \tag{3}$$

where $\mathbf{C} \in \mathbb{C}^{M,M}$ denotes the mutual coupling matrix (MCM) of the UCA, which describes how the received signal changes as a result of mutual coupling.

Thus the output of the array described by (2) will become

$$\mathbf{x}(t) = \mathbf{C}\mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \in \mathbb{C}^{M,1}. \tag{4}$$

Due to the circular symmetry of a UCA, the MCM $\mathbf{C}$ exhibits a complex symmetric circular Toeplitz structure [13]. Denote the first row of $\mathbf{C}$ by $\tilde{\mathbf{c}}$, which can be expressed as

$$\tilde{\mathbf{c}} = [c_1, c_2, \cdots, c_L, c_{L-1}, \cdots, c_3, c_2] \in \mathbb{C}^{1,M} \tag{5}$$

when the sensor number M is even with $L = M/2 + 1$, and

$$\tilde{\mathbf{c}} = [c_1, c_2, \cdots, c_L, c_L, \cdots, c_3, c_2] \in \mathbb{C}^{1,M} \tag{6}$$

when M is odd with $L = (M + 1)/2$. Thus, the MCM can be given by

$$\mathbf{C} = \text{toeplitz}(\tilde{\mathbf{c}}, \tilde{\mathbf{c}}) \in \mathbb{C}^{M,M}, \tag{7}$$

where toeplitz$(\bullet, \bullet)$ denotes the symmetric Toeplitz matrix.

From (5)-(7), it's easy to infer that there are only $L = \lfloor M/2 + 1 \rfloor$ unknown mutual coupling coefficients in MCM. Denote these unknown coefficients as a mutual coupling vector

$$\mathbf{c} = [c_1, c_2, \ldots, c_L]^T \in \mathbb{C}^{L,1}. \tag{8}$$

Since the mutual coupling coefficient between two different sensors is inversely proportional to their distance, the relationship of the elements of $\mathbf{c}$ satisfies

$$1 = c_1 > |c_2| > \ldots > |c_L| \geq 0. \tag{9}$$

In the following discussion, it is supposed that the impinging signals are uncorrelated with each other and independent with the noises. It is also assumed that the noises $\mathbf{n}(t)$ are spatially white Gaussian random processes with zero mean and σ_n^2 variance. Then the spatial covariance matrix of the array output vector $\mathbf{x}(t)$ can be expressed as

$$\begin{aligned} \mathbf{R}_x &= E\left[\mathbf{x}(t)\mathbf{x}^H(t)\right] \\ &= \mathbf{C}\mathbf{A}E\left[\mathbf{s}(t)\mathbf{s}^H(t)\right]\mathbf{A}^H\mathbf{C}^H + E\left[\mathbf{n}(t)\mathbf{n}^H(t)\right] \\ &= \mathbf{C}\mathbf{A}\mathbf{R}_s\mathbf{A}^H\mathbf{C}^H + \sigma_n^2\mathbf{I} \in \mathbb{C}^{M,M}, \end{aligned} \tag{10}$$

where H and E denote complex conjugate transpose and expectation, respectively.

Thus, the problem addressed here is to simultaneously estimates the array MCM and the DOAs using the array output vector $\mathbf{x}(t)$ and its spatial covariance matrix $\mathbf{R}_x$.

III. Proposed auto-calibration algorithm

A. Subspace-based DOA estimation

Performing eigen-decomposition on the array output covariance matrix $\mathbf{R}_x$, and letting λ_m and $\mathbf{e}_m$ be the eigenvalues and corresponding eigenvectors, the matrix can be written as

$$\mathbf{R}_x = \sum_{m=1}^{M} \lambda_m \mathbf{e}_m \mathbf{e}_m^H = \mathbf{E}_s\mathbf{\Lambda}_s\mathbf{E}_s^H + \mathbf{E}_n\mathbf{\Lambda}_n\mathbf{E}_n^H \in \mathbb{C}^{M,M}, \tag{11}$$

where $\mathbf{E}_s = [\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_K] \in \mathbb{C}^{M,K}$ contains the K principal eigenvectors corresponding to the K maximum eigenvalues as the signal subspace, and $\mathbf{E}_n \in \mathbb{C}^{M,M-K}$ contains the rest $(M-K)$ eigenvectors as the noise subspace [2].

Ideally, signal subspace spans the same space with the array manifold matrix, i.e. span$\{\mathbf{E}_s\}$ = span$\{\mathbf{C}\mathbf{A}\}$, and because $\mathbf{E}_s$ and $\mathbf{E}_n$ are orthogonal, it holds that span$\{\mathbf{E}_n\} \perp$ span$\{\mathbf{C}\mathbf{A}\}$.

So the DOAs and unknown mutual coupling can be estimated jointly by minimizing the cost function

$$J = \| \mathbf{E}_n^H\mathbf{C}\mathbf{A} \|_F^2 = \sum_{k=1}^{K} \| \mathbf{E}_n^H\mathbf{C}\mathbf{a}(\varphi_k) \|^2, \tag{12}$$

where $\| \bullet \|_F$ and $\| \bullet \|$ denote the Frobenius matrix norm and the vector 2-norm, respectively.

B. Auto-calibration algorithm

Based on the above analysis, the auto-calibration problem addressed in this letter has been formulated as an optimization problem described by (12). However, estimating a matrix directly is not an easy task since it possesses too many parameters. By taking advantage of the symmetric circular property of the MCM, we can transform it to a mutual coupling vector as [13]

$$\mathbf{C}\mathbf{a}(\varphi) = \mathbf{T}[\mathbf{a}(\varphi)]\mathbf{c} \in \mathbb{C}^{M,1}, \tag{13}$$

where $\mathbf{c} \in \mathbb{C}^{L,1}$ is referred to (8), $\mathbf{T}[\mathbf{a}(\varphi)] \in \mathbb{C}^{M,L}$ is the transform matrix defined as the sum of the following four matrices:

$$[\mathbf{T}_1]_{ij} = \begin{cases} \mathbf{a}(\varphi)_{i+j-1} & i + j \leq M + 1 \\ 0 & \text{otherwise} \end{cases} \tag{14a}$$

$$[\mathbf{T}_2]_{ij} = \begin{cases} \mathbf{a}(\varphi)_{i-j+1} & i \geq j \geq 2 \\ 0 & \text{otherwise} \end{cases} \tag{14b}$$

$$[\mathbf{T}_3]_{ij} = \begin{cases} \mathbf{a}(\varphi)_{M+1+i-j} & i < j \leq q \\ 0 & \text{otherwise} \end{cases} \tag{14c}$$

$$[\mathbf{T}_4]_{ij} = \begin{cases} \mathbf{a}(\varphi)_{i+j-M-1} & 2 \leq j \leq q, i + j \geq M + 2 \\ 0 & \text{otherwise,} \end{cases} \tag{14d}$$

where $q = \lfloor (M + 1)/2 \rfloor$. Plugging (13) into the cost function (12) yields

$$J = \sum_{k=1}^{K} \mathbf{c}^H\mathbf{T}^H[\mathbf{a}(\varphi_k)]\mathbf{E}_n\mathbf{E}_n^H\mathbf{T}[\mathbf{a}(\varphi_k)]\mathbf{c} = \mathbf{c}^H\mathbf{Q}(\varphi)\mathbf{c}, \tag{15}$$

where $\mathbf{Q}(\varphi)$ is a Hermitian matrix defined as

$$\mathbf{Q}(\varphi) = \sum_{k=1}^{K} \mathbf{T}^H[\mathbf{a}(\varphi_k)]\mathbf{E}_n\mathbf{E}_n^H\mathbf{T}[\mathbf{a}(\varphi_k)] \in \mathbb{C}^{L,L}. \tag{16}$$

Since $\mathbf{Q}(\varphi)$ is independent of $\mathbf{c}$, the auto-calibration of mutual coupling has become a quadratic minimization problem denoted as

$$(\{\varphi_k\}_{k=1}^K, \mathbf{c}) = \arg\min_{\{\varphi_k\}_{k=1}^K, \mathbf{c}} J = \arg\min_{\{\varphi_k\}_{k=1}^K, \mathbf{c}} \mathbf{c}^H \mathbf{Q}(\varphi)\mathbf{c}. \quad (17)$$

In general, the solution of this optimization problem requires a constraint to avoid the trivial solution. A norm constraint $\| \mathbf{c} \| = 1$ or a linear constraint such as $[\mathbf{c}]_1 = 1$ can be used depending on the application. Some solutions have been given in [16] and [17], respectively. However, there are two main shortcomings associated. One is that they have used the prior knowledge of the nonzero element number ξ in the mutual coupling vector, which is often not known exactly. The other is that serious ambiguity problems exist during the estimation process, which has been analysed in [18]. The ambiguity is mainly caused by the singularity or rank reduction of matrix $\mathbf{Q}(\varphi)$ at some ambiguous angles. In addition, the ambiguity is also determined by the choice of ξ.

Therefore, a more generalized and accurate method is proposed in this letter. First, given an initial value of vector $\mathbf{c}$, the problem of DOA estimation will reduce to the standard MUSIC algorithm with a one-dimensional peaks searching of

$$P(\varphi) = J^{-1} = (\mathbf{c}^H \mathbf{Q}(\varphi)\mathbf{c})^{-1}. \quad (18)$$

Then update the value of $\mathbf{c}$ by solving (17) using the currently estimated DOAs $\{\varphi_k\}_{k=1}^K$ under the constraint $\| \mathbf{c} \| = 1$. Thus, the above joint minimization can be performed iteratively, updating $\mathbf{c}$ and $\{\varphi_k\}_{k=1}^K$ alternatingly until convergence.

Summarizing the above proposed auto-calibration algorithm as follows:

Init $\mathbf{c}^{(0)} = [1, 0, \ldots, 0]^T \in \mathbb{C}^{L,1}$ or any recently calibrated or measured value, set $l = 0$.

1) Search for the K highest peaks of the spatial spectrum denoted by (18) with $\mathbf{c} = \mathbf{c}^{(0)}$, by using function "findpeaks" defined in Matlab. These peaks correspond to the newly estimated DOAs $\{\varphi_k^{(l)}\}_{k=1}^K$. Compute $J^{(l)}$ according to (15).
2) **Repeat**
3) Perform (17) with $\varphi = \{\varphi_k^{(l)}\}_{k=1}^K$ under the constraint of $\| \mathbf{c} \| = 1$. The result of this minimization problem is given by

$$\mathbf{c}^{(l+1)} = \mathbf{e}_{\min}\{\mathbf{Q}^{(l)}(\varphi)\}, \quad (19)$$

where $\mathbf{Q}^{(l)}(\varphi) = \mathbf{Q}(\varphi)|_{\varphi=\{\varphi_k^{(l)}\}_{k=1}^K}$, and $\mathbf{e}_{\min}\{\mathbf{Q}^{(l)}(\varphi)\}$ denotes the eigenvector corresponding to the smallest eigenvalue of matrix $\mathbf{Q}^{(l)}(\varphi)$.
4) Normalize $\mathbf{c}^{(l+1)}$ with respect to its first element $[\mathbf{c}^{(l+1)}]_1$.
5) Substitute $\mathbf{c}^{(l+1)}$ into (18) to search for the K highest peaks which corresponds to DOAs $\{\varphi_k^{(l+1)}\}_{k=1}^K$. Note that this choice of $\{\varphi_k^{(l+1)}\}_{k=1}^K$ minimizes $J^{(l+1)}$ for given $\mathbf{c}^{(l+1)}$.
6) $l \leftarrow l + 1$.
7) **Until** convergence, i.e. $J^{(l-1)} - J^{(l)} \leq \delta$, where δ is the threshold of convergence.

The success of peak searching above is mainly determined by the SNR and the magnitude of mutual coupling. Since our discussion focuses on the latter, a relatively high SNR is considered in this letter.

Compared with the algorithms in [16] and [17], the above method seldom encounters ambiguity problems. This is because the quadratic minimization problem expressed by (17) is calculated only with the K newly estimated DOAs instead of all possible direction angles, so that those ambiguous angles which makes matrix $\mathbf{Q}(\varphi)$ rank reduction will be avoided. Additionally, this method does not require any prior knowledge of mutual coupling, and can estimate DOAs and mutual coupling accurately no matter whether $\mathbf{c}$ contains zero elements or not.

Finally, a necessary but not sufficient identifiability condition of the proposed algorithm is given without proof:

$$K \leq \lfloor (M-1)/2 \rfloor. \quad (20)$$

IV. Simulation experiments

In this section, we present some computer simulations to illustrate the estimation results of the proposed algorithm. Some experiments are also made to compare the performance of the proposed algorithm with that in [16].

Consider a 7-sensor UCA of radius $r = 0.6\lambda$, then the number of unknown coupling coefficients is $L = \lfloor M/2+1 \rfloor = 4$. Assuming $\mathbf{c} = \mathbf{c}_1 = [1, 0.6325 + 0.3946j, 0.3514 + 0.2192j, 0]^T$. Three equal-power uncorrelated sources with SNR = 20dB impinge on the array from $-30°$, $25°$ and $60°$, respectively. The number of collected snapshots N is 500. Experiment results are depicted in Fig. 1. It is shown that both the proposed method and method in [16] can successfully estimate the DOAs since the two spatial spectrum curves share very sharp peaks at the three correct angles. However, the peaks of the uncalibrated curve are not sharp and deviate from their true positions, so it is not able to estimate the DOAs without calibration.

Then, change one of the simulation conditions, i.e. $\mathbf{c} = \mathbf{c}_2 = [1, 0.6325 + 0.3946j, 0.3514 + 0.2192j, 0.2816 + 0.1757j]^T$. For method in [16], some ambiguous peaks such as $-154.8°$ and $177.9°$ appear except for the three favorable ones, as shown in Fig. 2. Whereas our proposed method is very robust and performs as well as before. Again, the uncalibrated method keeps poor to estimate DOAs. It is noted that this choice of $\mathbf{c}$ is actually more close to reality, because the mutual coupling between two relatively far away sensors, though small, is not zero, and it is usually hard to tell the threshold of zero mutual coupling.

Next, set the mutual coupling vector back to $\mathbf{c}_1$ and change the sensor number of the UCA to 6. Simulation results are seen in Fig. 3. Ambiguity problems of the method in [16] emerge again, and the proposed method significantly outperforms the method in [16]. The uncalibrated method is still unable to estimate DOAs due to the array mutual coupling.

The last simulation considers the same scenario as the first one, i.e. $M = 7$ and $\mathbf{c} = \mathbf{c}_1$, where method in [16] does not possess ambiguity problems. 200 Monte Carlo experiments are performed to evaluate the statistical performance of the proposed algorithm. In each experiment, 500 snapshots of data are collected and the SNR ranges from 5 dB to 40 dB. Fig. 4 shows the estimated RMSE of the signal from $25°$ and the corresponding Cramer-Rao lower bound (CRB). It can be

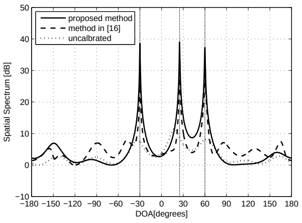

Fig. 1. Comparison of spatial spectrum for a 7-sensor UCA with $\mathbf{c} = \mathbf{c}_1$.

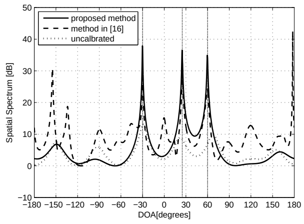

Fig. 2. Comparison of spatial spectrum for a 7-sensor UCA with $\mathbf{c} = \mathbf{c}_2$.

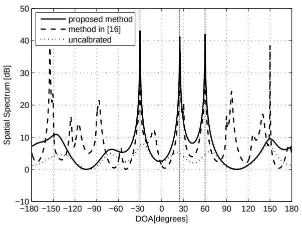

Fig. 3. Comparison of spatial spectrum for a 6-sensor UCA with $\mathbf{c} = \mathbf{c}_1$.

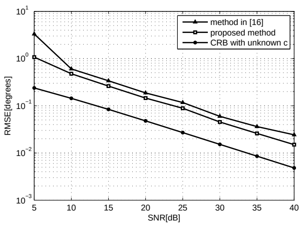

Fig. 4. RMSE of the DOA estimates and the corresponding CRB versus SNR.

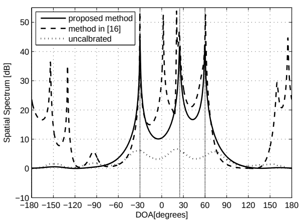

Fig. 5. Spatial spectrum for a 7-sensor UCA with $r = 0.5\lambda$ and $\mathbf{c} = \mathbf{c}_3$.

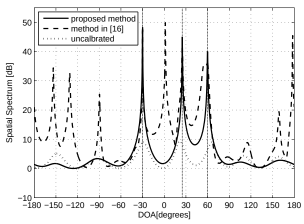

Fig. 6. Spatial spectrum for a 7-sensor UCA with $r = 0.7\lambda$ and $\mathbf{c} = \mathbf{c}_4$.

seen clearly that the proposed iterative method is efficient and achieves higher estimated accuracy.

It should be noted that the above coupling coefficients are selected by referring to [17], [19], and also determined by the inverse relationship with their sensors' distances. For smaller coupling coefficients, the proposed algorithm certainly works well. In essence, the performance of the algorithm is determined by the deviation between the available initial coupling value and the actual one. Specifically, the larger the deviation is, the more seriously the performance degrades. So the proposed method is not suitable for very large mutual coupling without a proper initial value of $\mathbf{c}$, since in addition to the performance degradation of peak searching, the iterative process may also converge to a local minimum of the cost function J.

At last, to demonstrate the performance of the proposed algorithm further, two simulations with different UCA sizes and related coupling coefficients are shown here. In Fig. 5, $r = 0.5\lambda$ and $\mathbf{c} = \mathbf{c}_3 = [1, 0.7 + 0.44j, 0.39 + 0.24j, 0.31 + 0.2j]^T$, while in Fig. 6, $r = 0.7\lambda$ and $\mathbf{c} = \mathbf{c}_4 = [1, 0.54 + 0.34j, 0.3 + 0.19j, 0.24 + 0.15j]^T$. Both of the simulation results have shown the superior performance of our method.

V. Conclusion

In conclusion, we present an iterative auto-calibration method to jointly estimate the DOA and unknown mutual coupling for a uniform circular array. This method is based on subspace theory and uses the special structure of mutual coupling matrix. While it does not require any prior knowledge of nonzero element number in mutual coupling vector and suffers from no ambiguity problems. Computer simulations have demonstrated the robustness, effectiveness and higher estimated accuracy of the proposed algorithm. In the future work, we might put our emphasis on some experimental measurements to validate our algorithm further.

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