ROBUST MINIMUN VARIANCE BEAMFORMING APPLIED TO ULTRASOUND IMAGING IN THE PRESENCE OF PHASE ABERRATION

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Minimum variance (MV) adaptive beamforming has been applied in medical ultrasound imaging to improve lateral resolution, but standard MV method is quite sensitive to the perturbation of the received signal. Due to the inhomogeneity of human tissues, sound speed varies and there exists a certain amount of time-delay error for each channel which is referred to as phase aberration. A robust minimum variance adaptive beamforming method was proposed in order to avoid significant degradation under more practical circumstances. The sample covariance matrix was reconstructed with a forward-backward method. And the steering vector was estimated via maximizing the output beamformer power, which was turned into a quadratic convex optimization problem with proper constraints. The simulations with Field II showed that the proposed method was robust in the presence of phase aberration and could maintain its performance for improving imaging quality.

Keywords: Ultrasound imaging; Adaptive beamforming; Phase aberration

1. INTRODUCTION

Minimum variance (MV) adaptive beamforming method, also well known as Capon's method, has been successfully applied in medical ultrasound imaging to improve lateral resolution [1]. Compared with the conventional delay-and-sum (DAS) beamforming method, MV beamforming can reduce aperture size, increase frame rate and penetration depth without sacrificing imaging quality [2]. However, such adaptive beamforming methods have high sensitivity to model assumptions. Standard MV method is quite sensitive to the perturbation of covariance matrix and the mismatch of signal steering vector, which limits its application in more complicated situations.

There is a certain amount of time-delay error for each channel in most of ultrasound imaging systems because of the inhomogeneous distribution of sound speed. This problem is referred to as phase aberration and has become a main source of image degradation [3]. Under such circumstance, the performance of MV adaptive beamforming method is highly affected. Though it has been validated that MV method still outperforms DAS method in the presence of phase aberration [4], its performance drops much more significantly considering its remarkable improvement of imaging quality when applied in ideal situations.

More robust signal processing techniques should be developed in order to maintain its good performance under more practical circumstances. A steering vector estimation method combined with forward-backward averaging technique was proposed in this paper and it could improve imaging quality in the presence of phase aberration.

2. METHDOLOGY

2.1. MV beamforming method in medical ultrasound

For an ultrasound imaging system with an aperture consisted of M elements, the received signal $\mathbf{x}(k)$ can be decomposed as the following equation:

$$\mathbf{x}(k)=\mathbf{s}(k)+\mathbf{i}(k)+\mathbf{n}(k)=\mathbf{a}s(k)+\mathbf{v}(k)\,, \qquad (1)$$

where useful information about the target is in $\mathbf{s}(k)$. $\mathbf{a}$ is the steering vector which defines the phase of signal received from each channel and $s(k)$ contains the waveform information. $\mathbf{i}(k)$ and $\mathbf{n}(k)$ is the interference vector and noise vector respectively, the combination of which is defined as $\mathbf{v}(k)$.

The output signal after beamforming is formulated as following:

$$y(k)=\mathbf{w}^H(k)\mathbf{x}_\mathbf{d}(k)=\sum_{i=1}^{M} w^*_i(k)x_i(k-\Delta_i)\,, \qquad (2)$$

where $\mathbf{x}_\mathbf{d}$ is the time-delayed version of the received signal and $\mathbf{w}(k)$ is the weight vector applied to the aperture. $x_i(k)$ is the received signal from the ith element and time-delay parameter Δ_i is calculated according to the presumed sound speed and the geometrical position

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of the target and each element of the transducer.

The weight vector **w** is independent of the received signal in conventional DAS beamforming method, however, in MV adaptive beamforming it is computed according to the received data and varies with different time indexes. The calculation procedure can be described mathematically in the following:

$$\begin{aligned} &\min_{\mathbf{w}(k)} \quad \mathbf{w}^H(k)\mathbf{R}_\mathrm{v}(k)\mathbf{w}(k) \\ &\text{s.t.} \qquad \mathbf{w}^H(k)\mathbf{a} = 1 \end{aligned} \quad , \tag{3}$$

where $\mathbf{R}_\mathrm{v}(k)$ is the interference-plus-noise covariance matrix and **a** is the steering vector. The solution to the above optimization problem is

$$\mathbf{w}_\mathrm{MV}(k) = \frac{\mathbf{R}_\mathrm{v}^{-1}(k)\mathbf{a}}{\mathbf{a}^H\mathbf{R}_\mathrm{v}^{-1}(k)\mathbf{a}} \ . \tag{4}$$

The calculated $\mathbf{w}_\mathrm{MV}$ is then applied in Eq. (2) to improve imaging quality.

2.2. Covariance matrix reconstruction

The interference-plus-noise covariance matrix is unavailable in most cases and it is usually replaced by the sample covariance matrix:

$$\mathbf{R}(k) = \mathbf{x_d}(k)\mathbf{x_d}^H(k) \ . \tag{5}$$

The received signal in medical ultrasound is highly coherent which is not appropriate for the Capon's assumption. Therefore a decorrelation technique called spatial smoothing should be applied [5]:

$$\hat{\mathbf{R}}(k) = \frac{1}{M-L+1}\sum_{l=1}^{M-L+1} \mathbf{x_d}^l(k)\mathbf{x_d}^l(k)^H \ , \tag{6}$$

where L is the subarray length in the spatial smoothing procedure and $\mathbf{x_d}^l(k) = [x_\mathrm{d}^l(k), x_\mathrm{d}^{l+1}(k), ..., x_\mathrm{d}^{l+L-1}(k)]^\mathrm{T}$, which is named as forward-only estimate.

It is referred to as backward estimate accordingly if the time-delayed version of received data vector is taken as $\tilde{\mathbf{x}}_\mathbf{d}^l(k) = [x_\mathrm{d}^{M-l+1}(k), x_\mathrm{d}^{M-l}(k), ..., x_\mathrm{d}^{M-l-L+2}(k)]^\mathrm{T}$. And the covariance matrix is reconstructed via forward-backward estimation:

$$\hat{\mathbf{R}}_\mathrm{FB}(k) = \frac{1}{2}(\hat{\mathbf{R}}(k) + \tilde{\mathbf{R}}(k)) \ , \tag{7}$$

where $\hat{\mathbf{R}}(k)$ and $\tilde{\mathbf{R}}(k)$ is estimated with forward and backward method respectively.

The reconstructed covariance matrix $\hat{\mathbf{R}}_\mathrm{FB}(k)$ is used in the proposed robust MV adaptive beamforming method to improve imaging quality. Compared with the commonly used diagonal loading method, it can reduce the sensitivity to the aberrated signal without seeking an optimal diagonal loading factor [6].

2.3. Steering vector estimation

The steering vector in Eq. (3) is simply a vector of ones in the assumption that all the received data is aligned. However this is not the case in the presence of phase aberration and there exists a certain amount of error between the actual steering vector and the presumed one. The actual steering vector will be estimated via the covariance matrix reconstructed above to compensate the performance degradation brought by phase aberration in more practical situations.

In the proposed method the steering vector is estimated via maximizing the output beamformer power:

$$P_o = \frac{1}{\mathbf{a}^H\hat{\mathbf{R}}_\mathrm{FB}^{-1}(k)\mathbf{a}} \ , \tag{8}$$

where $\hat{\mathbf{R}}_\mathrm{FB}^{-1}$ is the inverse of the covariance matrix reconstructed in Eq. (7). The estimated steering vector **a** can be decomposed into two parts [7]: the presumed steering vector $\bar{\mathbf{a}}$ and the mismatch vector $\mathbf{e}_\perp$ which is orthogonal to $\bar{\mathbf{a}}$. Because the parallel part of the mismatch vector is only a scaled copy of $\bar{\mathbf{a}}$ which does not affect the final beamformer output, the optimization problem can be simplified into:

$$\begin{aligned} &\min_{\mathbf{e}_\perp} \qquad (\bar{\mathbf{a}} + \mathbf{e}_\perp)^H\hat{\mathbf{R}}_\mathrm{FB}^{-1}(\bar{\mathbf{a}} + \mathbf{e}_\perp) \\ &\text{s.t.} \qquad \bar{\mathbf{a}}^H\mathbf{e}_\perp = 0 \\ &\qquad (\bar{\mathbf{a}} + \mathbf{e}_\perp)^H\hat{\mathbf{R}}_\mathrm{FB}(\bar{\mathbf{a}} + \mathbf{e}_\perp) \le \bar{\mathbf{a}}^H\hat{\mathbf{R}}_\mathrm{FB}\bar{\mathbf{a}} \end{aligned} \ . \tag{9}$$

The second constraint in Eq. (9) is used to prevent the estimated steering vector from converging to the interference region which will degrade the performance of MV beamformer [8].

Since the reconstructed covariance matrix is positive-definite, the optimization problem in Eq. (9) is turned into a quadratically constrained quadratic programming problem, which can be feasibly solved via CVX, a package for specifying and solving convex problems [9].

The proposed robust adaptive weight vector was finally computed with the above reconstructed sample covariance matrix and estimated steering vector to achieve more robust performance against phase aberration.

3. SIMULATION

The proposed method was verified via simulations with Field II program [10] and a near-field phase screen model was used to describe phase aberration [11].

A linear transducer was used in the simulation and the transmitting pulse is two periods sine wave modulated by the Hanning function. The transmit focus was fixed at 60mm depth and dynamic focusing was used in the reception procedure. Other specific parameters for the simulation are listed in Table 1.

Table 1. Parameters used for the simulation

Parameter	*Value*
Sampling frequency (MHz)	40
Center frequency (MHz)	3.5
Element height (mm)	10
Element width (mm)	0.12
Element pitch (mm)	0.22
Dynamic range (dB)	60
Element number	64
A-line number	127

Two columns of point targets were first simulated, lateral location of which was -2mm and 2mm respectively. And axially they lay from 50mm to 70mm with 5mm separated from each other. The B-mode imaging results and corresponding beam pattern taken at 60mm depth are illustrated in Figure 1 and Figure 2 respectively.

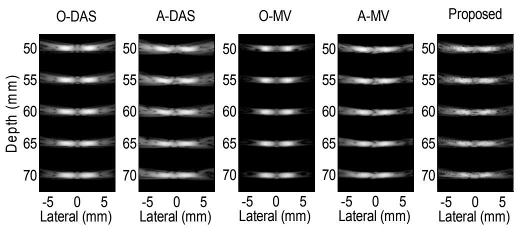

Figure 1. Simulation results of B-mode imaging for point targets

From left to right, Fig. 1 illustrates the conventional DAS beamforming method with original signal (O-DAS), DAS method with aberrated signal (A-DAS), MV beamforming method with original signal (O-MV), MV method with aberrated signal (A-MV) and the proposed method respectively.

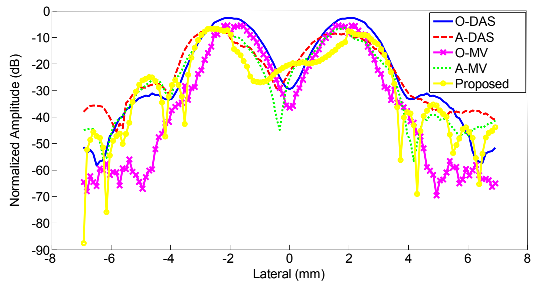

Figure 2. Beam pattern of point targets taken at 60mm depth

From the above results, we can tell that the proposed method could narrow the mainlobe compared to the standard MV method with aberrated signal, though the side-lobes are still obvious considering the good performance in ideal situation.

The imaging results of simulation on an anechoic cyst target are illustrated in Figure 3, where the center of the speckle was an anechoic cyst with a radius of 4mm. It lay at 60mm depth with a lateral location at 0mm and random point targets were distributed around the cyst.

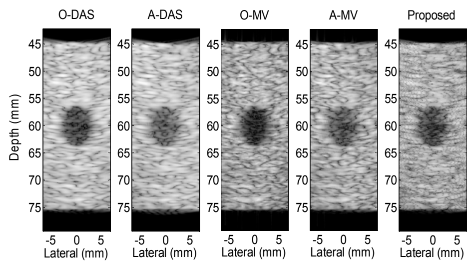

Figure 3. Simulation results of B-mode imaging for an anechoic cyst target

To quantitively analyze the imaging results, the contrast ratio (CR) and contrast-to-noise ratio (CNR) [12] were calculated according to Eq. (10) and Eq. (11):

$$CR = \left| \langle S_o \rangle - \langle S_i \rangle \right|, \tag{10}$$

$$CNR = \frac{CR}{\sqrt{\sigma_i^2 + \sigma_o^2}}, \tag{11}$$

where $\langle S_i \rangle$, $\langle S_o \rangle$, σ_i and σ_o is the mean intensity and standard deviation inside and outside the cyst region respectively. The results are listed in Table 2.

Table 2. Numerical results of simulation on the cyst

Method	$\langle S_i \rangle$ *(dB)*	$\langle S_o \rangle$ *(dB)*	$\sqrt{\sigma_i^2+\sigma_o^2}$ *(dB)*	*CR (dB)*	*CNR*

O-DAS	-39.51	-15.07	10.02	24.44	2.44
A-DAS	-34.38	-15.48	8.60	18.90	2.20
O-MV	-45.57	-21.58	11.32	24.00	2.12
A-MV	-38.24	-19.98	9.51	18.27	1.92
Proposed	-41.36	-20.32	12.48	21.05	1.69

Compared with DAS beamformer, MV adaptive methods have a lower contrast-to-noise ratio because the improvement in lateral resolution results in more fluctuations in background. The proposed method outperforms the standard MV and DAS beamforming 2.78 dB and 2.15 dB respectively in contrast ratio in the presence of phase aberration. But one of the side-effects brought at the same time is the higher level of noise in background, which reflects in a lower contrast-to-noise ratio.

4. CONCLUSION

The proposed method is robust in the presence of phase aberration in medical ultrasound imaging and no prior knowledge about the phase error is needed. Though the proposed method can maintain its performance to some extent, its improvement of imaging quality is still limited because the phase error is not corrected at all. Meanwhile, to solve the convex optimization problem in steering vector estimation demands a high computational cost. More flexible algorithms need to be developed in order to make the method implemented in practical use. And the signal error model of phase aberration will be also furthered studied. The combination of robust adaptive beamforming method and phase aberration correction algorithm is promising, which can bring much more improvements for imaging quality in medical ultrasound.

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