Wideband DOA estimation based on block FOCUSS with limited samples

Jiawei Zhang[1], Nan Hu[2], Ming Bao[1*], Xiaodong Li[1] and Wei He[3]
Lab of Communication Acoustics, Institute of Acoustics, Chinese Academy of Sciences, Beijing, China[1]
Department of Electronic Engineering and Information Science, University of Science and Technology of China, Hefei, China[2]
Key Laboratory of Wireless Sensor Network & Communication, Shanghai Institute of Microsystem and Information Technology, Chinese Academy of Sciences, Shanghai, China[3]

***Abstract*—Whittaker-Shannon interpolation formula instead of narrowband filter (NF) is used to interpolate the signal more precisely in wideband direction-of-arrival (DOA) estimation problem when the number of samples is highly limited. A novel algorithm named block FOCal Underdetermined System Solver (BFOCUSS) is proposed to solve this problem. The simulation results validate the superior performance compared with the other algorithms.**

Keywords — direction-of-arrival; block sparsity; block FOCUSS; Whittaker-Shannon interpolation formula

I. Introduction

Wideband direction-of-arrival (DOA) estimation has been widely used in various active research areas hence many kinds of algorithms have been proposed to address this problem. The covariance matrix of the array output data are utilized by those subspace-based wideband DOA estimation algorithms such as incoherent signal subspace methods (ISSM) [1] and coherent signal subspace methods (CSSM) [2], to acquire super-resolution performance. Since these subspace-based methods use a bank of narrowband filters (NF) to obtain data which are highly correlated in time, they need a sufficient large number of samples to yield accurate estimates at each frequency bin. In addition, ISSM cannot deal with coherent signals, while CSSM needs the preliminary estimation for focusing procedure.

Recently, sparse signal representation has been introduced in DOA estimation and a number of sparse recovery algorithms have been proposed. Solving multiple measurement vectors (MMV) problem is used to estimate DOA by utilizing the joint sparsity structure of the vectors. In [3], MMV Basic Matching Pursuit (M-BMP) and MMV Orthogonal Matching Pursuit (M-OMP) were proposed. M-BMP and M-OMP can find a sparse solution by sequentially selecting columns which match residual matrix best. FOCal Underdetermined System Solver (FOCUSS) which can acquire sparse solutions iteratively was proposed in [4] and [5]. Since the original FOCUSS can only estimate DOA by single snapshot, the poor estimation performance under the low signal-to-noise ratio (SNR) condition blocks its application. As a result, MMV FOCUSS (M-FOCUSS) [3] was proposed based on multiple snapshots. ℓ_1-SVD [6], which uses the SVD step of the subspace algorithm together with ℓ_1-norm minimization, can estimate closely spaced correlated narrowband sources with high accuracy. However, the spatial spectrums from different frequencies are aliased in wideband DOA estimation problem, since such a wideband method does not consider the joint sparsity property. To address this property, wideband DOA estimation problem was casted as a basis pursuit de-noising problem and solved by homotopy approach in [7]. In [8], a mixed $\ell_{2,0}$-norm approximation, which utilizes jointly sparse property and uses Gaussian function instead of ℓ_1-norm to approximate ℓ_0-norm in DOA estimation problem, was proposed. Recently, many researchers pay a lot of attention on sparse Bayesian learning (SBL). MMV Sparse Bayesian Learning (MSBL) was introduced in [9] to deal with DOA problems. However, since NF is still involved in the sparse signal representation model, it does not perform well when the number of samples is highly limited.

However, wideband DOA estimation cannot be casted as a MMV problem, since every measurement vector in wideband DOA estimation does not share the same basis representative matrix. To solve this problem, in this paper, wideband DOA estimation is treated as a block sparsity reconstruction problem to utilize the joint sparsity property among different blocks. [10] proved that better results can be achieved if block sparsity rather than conventional sense is considered in the recovery of block-sparse signals, since additional structures are ignored in conventional methods. Block Orthogonal Matching Pursuit (BOMP) was also proposed in [10] to recover block-wise sparse signals, but the resolution of this algorithm is very low.

In this paper, Whittaker-Shannon interpolation formula [11] instead of NF is used to interpolate highly limited signals more precisely and turn wideband DOA estimation into a block sparsity reconstruction problem, then Block FOCUSS (BFOCUSS) is introduced to deal with this problem. The simulation results show the superior performance of BFOCUSS in wideband DOA estimation.

The rest of this paper is arranged as follows. In Section Ⅱ, wideband DOA estimation problem is reformulated by utilizing Whittaker-Shannon interpolation formula and then this problem is casted as a block-sparse signal reconstruction problem which is solved by the proposed BFOCUSS in Section Ⅲ. The simulation results show the advantages of this algorithm in Section Ⅳ and we draw some conclusions in Section Ⅴ.

II. PROBLEM FORMULATION

Consider a sensor array of M omnidirectional sensors is impinged by K wideband sources s_k, $k=0,1,\cdots,K-1$ from K distinct directions θ_k in far-field. The sampling rate of array is f which is two times of the single-side bandwidth B of wideband sources and every sensor obtains $2L+1$ samples.

The l th sample output of the m th sensor is

$$x_m(t_l)=\sum_{k=0}^{K-1} s_k(t_l-\tau_m(\theta_k))+n_m(t_l), \quad m=0,\cdots,M-1, \qquad l=-L,\cdots,L \tag{1}$$

where $\tau_m(\theta_k)$ is the time delay of k th signal in the m th sensor relative to the reference sensor, $n_m(t_l)$ is the noise and $t_l=l/f$.

Whittaker-Shannon interpolation formula [11] is used to interpolate source signals

$$s_k(t)=\sum_{n=-\infty}^{\infty}\varphi(n,tf)z_k(n),\ k=0,\cdots,K-1 \tag{2}$$

where $\varphi(n,tf)=\dfrac{\sin\pi(tf-n)}{\pi(tf-n)}$ and $z_k(n)=s_k(n/f)$.

To make the length of $s_k(t)$ finite, $-N\le n\le N$ is used to approximate source signals which makes (2) becomes

$$s_k(t)\approx\sum_{n=-N}^{N}\varphi(n,tf)z_k(n),\ k=0,\cdots,K-1 \tag{3}$$

Truncation error bound for Whittaker-Shannon interpolation formula was given in [12]. It shows that if there is sufficient number of $z_k(n)$ been used to interpolate wideband source signal $s_k(t)$, the truncation error can be ignored. Here we choose

$$N=2L. \tag{4}$$

Substituting (3) and (4) into (1), we obtain:

$$x_m(t_l)=\sum_{k=0}^{K-1}\sum_{n=-2L}^{2L}\varphi(n,(t_l-\tau_m(\theta_k))f)z_k(n)+n_m(t_l), \quad m=0,\cdots,M-1,\ l=-L,\cdots,L \tag{5}$$

Therefore, the output of all sensors in the array can be expressed as

$$\mathbf{X}=\mathbf{\Psi Z}+\mathbf{N}=\sum_{k=0}^{K-1}\mathbf{\Phi}(\theta_k)\mathbf{Z}_k+\mathbf{N} \tag{6}$$

where the outputs of all the sensors are rearranged as a vector: $\mathbf{X}=[\mathbf{X}_0^T,\cdots,\mathbf{X}_m^T,\cdots,\mathbf{X}_{M-1}^T]^T$, the output of the m th sensor is $\mathbf{X}_m=[x_m(t_{-L}),\cdots,x_m(t_l),\cdots,x_m(t_L)]^T$,

$\mathbf{\Psi}=[\mathbf{\Phi}(\theta_0),\cdots,\mathbf{\Phi}(\theta_k),\cdots,\mathbf{\Phi}(\theta_{K-1})]$,

$\mathbf{\Phi}(\theta_k)=[\mathbf{\Phi}_0^T(\theta_k),\cdots,\mathbf{\Phi}_m^T(\theta_k),\cdots,\mathbf{\Phi}_{M-1}^T(\theta_k)]^T$,

$$\mathbf{\Phi}_m(\theta_k)=\begin{pmatrix}\varphi(-2L,(t_{-L}-\tau_m(\theta_k))f) & \cdots & \varphi(2L,(t_{-L}-\tau_m(\theta_k))f)\\ \vdots & \ddots & \vdots\\ \varphi(-2L,(t_L-\tau_m(\theta_k))f) & \cdots & \varphi(2L,(t_L-\tau_m(\theta_k))f)\end{pmatrix},$$

$\mathbf{Z}=[\mathbf{Z}_0^T,\cdots,\mathbf{Z}_k^T,\cdots,\mathbf{Z}_{K-1}^T]^T$ is a vector which is rearranged by all the sources outputs, $\mathbf{Z}_k=[z_k(t_{-2L}),\cdots,z_k(t_l),\cdots,z_k(t_{2L})]^T$ is the output of the k th source and the noise is $\mathbf{N}=[n_0(t_{-L}),\cdots,n_0(t_L),\cdots,n_m(t_l),\cdots,n_{M-1}(t_{-L}),\cdots,n_{M-1}(t_L)]^T$.

Assume that $\mathbf{\Theta}=\{\tilde{\theta}_0,\cdots,\tilde{\theta}_p,\cdots,\tilde{\theta}_{P-1}\}$ is a dense direction grid which covers all the incident directions, hence (6) can be transformed to be:

$$\mathbf{X}=\mathbf{\Psi}\tilde{\mathbf{Z}}+\mathbf{N}=\sum_{p=0}^{P-1}\mathbf{\Phi}(\tilde{\theta}_p)\tilde{\mathbf{Z}}_p+\mathbf{N} \tag{7}$$

where $\tilde{\mathbf{Z}}_p=\mathbf{Z}_k$ if and only if $\tilde{\theta}_p=\theta_k$, otherwise $\tilde{\mathbf{Z}}_p=\mathbf{0}$.

III. WIDEBAND DOA ESTIMATION BASED ON BFOCUSS

The sparsity of (7) can be treated as block-wise, hence wideband DOA estimation can be formulated as a block-sparse signal reconstruction problem [8]:

$$\min J(\tilde{\mathbf{Z}})=\sum_{p=0}^{P-1} I(|\tilde{\mathbf{Z}}_p|_2>0) \quad s.t. |\mathbf{X}-\mathbf{\Psi}\tilde{\mathbf{Z}}|_2\le\varepsilon \tag{8}$$

where $I(\cdot)$ is an indicator function and ε denotes the bound of the error induced by the noise.

[10] mentioned that better results can be achieved if block sparsity rather than conventional sense is considered in the recovery of block-sparse signals, since additional structures are ignored in conventional methods. BOMP was proposed in [10] to solve block-sparse problem. However, the resolution of wideband DOA estimation by using BOPM is low. M-FOCUSS, which was proposed in [3], can be utilized to estimate narrowband DOAs with iteration procedures refining the low resolution initial estimate. Whereas, MMV model is unsuitable for wideband DOA estimation. Motivated by M-FOCUSS which utilizes joint sparsity property, we propose BFOCUSS to estimate wideband DOA.

Unlike M-FOCUSS weights $\tilde{Z}$ by every row (In M-FOCUSS, $\tilde{Z}$ is a matrix), BFOCUSS weights $\tilde{Z}$ by every block. The r th iteration of the proposed BFOCUSS algorithm is summarized as follows:

$$\mathbf{W}^{(r)}=diag(\mathbf{w}^{(r)}) \tag{9}$$

where $diag(\mathbf{w}^{(r)})$ is the diagonal matrix of vector $\mathbf{w}^{(r)}$, $\mathbf{w}^{(r)} = [\underbrace{w_0^{(r)},\cdots,w_0^{(r)}}_{4L+1},\cdots,\underbrace{w_{P-1}^{(r)},\cdots,w_{P-1}^{(r)}}_{4L+1}]$ and $w_p^{(r)} = \left\|\tilde{\mathbf{Z}}_p^{(r-1)}\right\|_2$

$$\mathbf{q}^{(r)} = \left(\mathbf{\Psi}\mathbf{W}^{(r)}\right)^{\dagger}\mathbf{X} \tag{10}$$

where $(\cdot)^{\dagger}$ denotes the Moore-Penrose pseudo-inverse.

The block-sparse solution $\tilde{\mathbf{Z}}$ at r th iteration can be calculated as:

$$\tilde{\mathbf{Z}}^{(r)} = \mathbf{W}^{(r)}\mathbf{q}^{(r)} \tag{11}$$

The iterative algorithm (9) ~ (11) can be interpreted as the following weighted minimum norm solution:

$$\begin{aligned} &\min_{\tilde{\mathbf{Z}}} \|\mathbf{q}\|_2 \\ &s.t.\ \tilde{\mathbf{Z}} = \mathbf{W}\mathbf{q} \\ &\quad \mathbf{X} = \tilde{\mathbf{\Psi}}\tilde{\mathbf{Z}} \end{aligned} \tag{12}$$

(10) is the solution of minimization procedure in (12) and weighted process in (11) makes sure that the solution of BFOCUSS approximate to the solution of the block-sparse signal reconstruction problem in (8).

In first iteration, $diag(\mathbf{w}^{(1)})$ is initialized as $(4L+1)P \times (4L+1)P$ identity matrix which will lead the first iteration to give a rough estimate of the sparse solution.

IV. Simulation Results

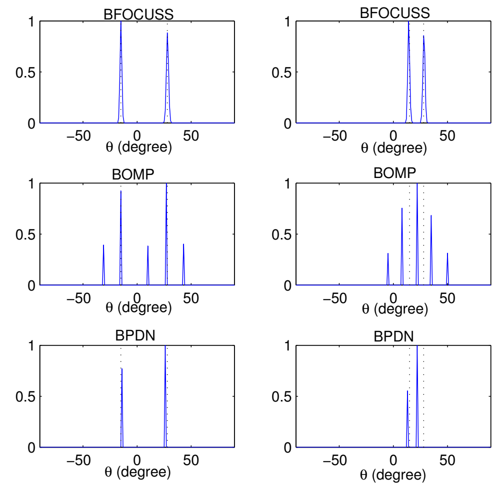
In this section, BFOCUSS is simulated, compared with other wideband DOA estimation algorithms such as BOMP [10] and BPDN [7]. Other algorithms are not considered because they do not perform well when the number of samples is highly limited (around 20 samples).

$K = 2$ uncorrelated wideband signals which cover the same frequency range $-B \sim B$ ($B = 500Hz$) are simulated to impinge on a uniform linear array (ULA) in far field. The distance between two nearby sensors is half of wavelength corresponding to the highest frequency. The sampling rate is $f = 2B$. White Gaussian noise is imposed on every sensor independently. $P = 181$ angles are considered in direction grid which means the resolution of the grid is $1°$.

In simulations, BFOCUSS and BOMP iterate 10 times and 5 times respectively and both of them formulate as block-wise signal problem by Whittaker-Shannon interpolation formula. BPDN uses a bank of NF to obtain data and the number of iteration is fixed at 2 which means the number of sources is known in prior in this method's simulation.

First of all, the estimation performances of BFOCUSS, BOMP and BPDN are compared. Then, the detection rate of BFOCUSS and BPDN are shown under the condition that SNR, number of sensors and samples are different.

In Fig. 1, $M = 9$ sensors are used, every sensor samples $2L+1 = 21$ data and SNR is 20dB. Two uncorrelated sources at $-15°$ and $28°$ are considered in Fig. 1 (a) and two uncorrelated sources in Fig. 1 (b) are at $15°$ and $28°$.



(a) (b)

Fig. 1. Comparison of BFOCUSS, BOMP and BPDN. (a) $\boldsymbol{\theta} = [-15°, 28°]$. (b) $\boldsymbol{\theta} = [15°, 28°]$

Fig. 1 (a) shows that these three algorithms can estimate DOAs accurately when sources are not closely spaced. However, when sources are closely spaced, e.g. Fig.1 (b), only BFOCUSS can distinguish two DOAs. In addition, BOMP produces some fake peaks that may influence the estimation result.

The following two simulations compare the detection rate of BFOCUSS and BPDN. There are 500 Monte Carlo trials tested in every condition. Two uncorrelated sources at $5°$ and $28°$ are considered. One successful detection is defined as the case when the biases of the both estimates are less than $3°$ which is around a quarter of beamwidth of these ULAs. BOMP is not considered here because its detection rate is too low when the sources are closely spaced.

In Fig. 2, $M = 7$ sensors are used and $2L+1 = 15, 21, 31$ samples ($L = 7, 10, 15$) are acquired in every experiment. The results show that the performances are improved in both BFOCUSS and BPDN, if SNR or number of samples is increased. Besides, the detection rate of BFOCUSS is shown to be higher than that of BPDN in these scenarios.

In Fig. 3, $2L+1 = 21$ samples ($L = 10$) are acquired, the number of sensors is $M = 5, 7, 9$ in every experiment. According to the result, the detection rate grows as the SNR and number of sensors is increased. What's more, it's shown that BFOCUSS has better performance than BPDN.

V. Conclusion

In this paper, wideband DOA estimation problem has been formulated as a block-sparse signal reconstruction problem by using Whittaker-Shannon interpolation formula. And then,

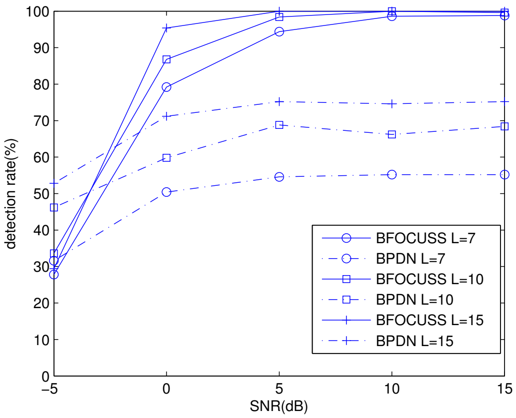


Figure 2. the detection rate of BFOCUSS and BPDN when the number of sensors is $M = 7$.

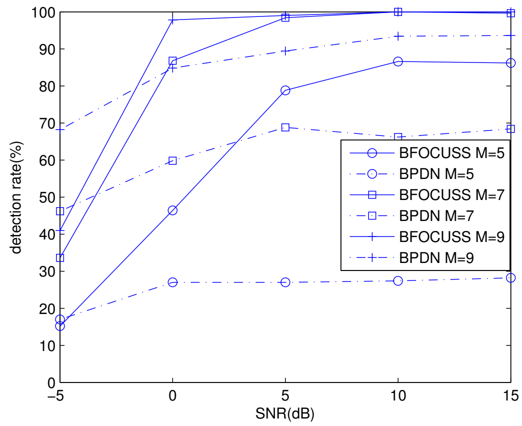


Figure 3. the detection rate of BFOCUSS and BPDN when the number of samples is $2L+1=21$.

BFOCUSS algorithm has been proposed to solve block-wise signal reconstruction problem. The simulation results have shown that BFOCUSS has better performance than other wideband DOA estimation algorithms when the number of samples is highly limited.

ACKNOWLEDGEMENT

This work is supported by the "Strategic Priority Research Program" of the Chinese Academy of Sciences (Grant No. XDA06020201) and the National Natural Science Foundation of China (Grant No. 11174316).

REFERENCES

[1] G.Su and M.Morf, "the signal subspace approach for multiple wide-band emitter location," *IEEE Transactions on Acoustic, Speech and Signal Processing*, vol. 31, no. 6, 1983, pp. 1502-1522.

[2] H.Wang and M.Kaveh, "Coherent signal-subspace processing for the detection and estimation of angles of arrival of multiple wide-band sources," *IEEE Transactions on Acoustic, Speech and Signal Processing*, vol.33, no.4, 1985, pp. 823-831.

[3] S.F.Cotter , B.D.Rao, K.Engan and K.Kreutz-Delgado, "Sparse Solution to Linear Inverse Problems With Multiple Measurement Vectors," *IEEE Transactions on Signal Processing*, vol. 53, no.7, 2005, pp. 2477-2488.

[4] I.F.Gorodnitsky and B.D.Rao, "Sparse Signal Reconstruction from Limited Data Using FOCUSS: A Re-weighted Minimum Norm Algorithm," *IEEE Transactions on Signal Processing*, vol. 45, no. 3, 1997, pp. 600-616.

[5] B.D.Rao and K.Kerutz-Delgado, "An affine scaling methodology for best basis selection," *IEEE Transactions on Signal Processing*, vol. 47, no. 1, 1999, pp. 187-200.

[6] D.Malioutov, M.Cetin and A.S.Willsky, "A Sparse Signal Reconstruction Persective for Source Localization With Sensor Arrays," *IEEE Transcations on Signal Processing*, vol. 53, no. 8, 2005, pp. 3010-3022.

[7] C.Liu and T.Chen, "Broadband Underwater Localization of Multiple Sources Using Basis Pursuit De-Noising," *IEEE Transactions on Signal Processing*, vol. 60, no. 4, 2011, pp. 1708-1717.

[8] M.M.Hyder and K.Mahata, "Direction-of-arrival estimation using a mixed $\ell_{2,0}$ -norm approximation," *IEEE Transcations on Signal Processing*, vol. 58, no.9, 2010, pp. 4646-4655.

[9] D.P.Wipf and B.D.Rao, "An empirical bayesian strategy for solving the simultaneous sparse approximation problem," *IEEE Transcations on Signal Processing*, vol. 55, no 7, 2007, pp. 3704-3716.

[10] Y.C.Eldar, P.Kuppinger and H.Bolcskei, "Block-Sparse Signals: Uncertainty Relations and Efficient Recovery," *IEEE Transactions on Signal Processing*, vol. 58, no. 6, 2010, pp. 3042-3054.

[11] C.E.Shannon, "Communication in the Presence of Noise," *in Proc. IRE*, vol. 37, 1949, pp. 10-21.

[12] K.Yao and J.B.Thomas, "On Truncation Error Bounds for Sampling Representations of Band-Limited Signals," *IEEE Transactions on Aerospace and Electronic Systems,* vol. AES-2, no. 6, 1966, pp. 640-647.